

Canonical Correlation between Human Capital and Socio – Economic Indices

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Abstract

This research reveals the association between human capital indices (mortality rate of children under 5 years, adult survival rate, Harmonized Test Scores (HTS) and expected years of school) and socio – economic indices (Life expectancy at birth, GDP per capita, and government expenditure on education). Using the canonical correlation analysis CCA, that assessing the association between the human capital indices as dependent set and socio-economic indices as predictor set. Data were from the World Bank (2018) for 108 countries. The Wilks's criteria show that the full model for all functions statistically significant; the full model explain 97.6% of the variance between the dependent and predictor sets. The first canonical correlation explicit a higher positive significant correlation 0.98 (P,0.000) between human capital and socio – economic indices. 74% of the human capital index variability is construed by the first canonical variate of socio-economic indices and 47% of the socio- economic indices' variability is construed by the first canonical variate of human capital indices in the model. Life expectancies at birth has the biggest contribution to the first socio – economic canonical variate among other predictors. The expected years of school and probability of survival to age 5 contribute more than the adult survival rate and Harmonized Test Scores (HTS) to human capital.

Keywords: Adult Survival Rate, Canonical Correlation Analysis, Structure Coefficient, Harmonized Test Scores, Human Capital, Life Expectancy at Birth, Mortality Rate.

Preamble:

Capital is a stock of dividends and return on investment for the progression in future. Human capital is the productivity of human resources which increases the salary as the productivity enhances. The productivity of human capital is correlated with abilities and lucrative information which including the knowledge, skills, and ability to perform tasks. Human capital includes education, training, skills, intelligence, and health. Investment on human resource in a country enhance the abilities which will increase the productivity. Inability to keep up with innovation depreciates the human capital in long run like other assets. In global economies, human capital usually tends to migrate, which known as human capital flight.

Economic growth and the per capita income development are in related strongly with the human capital growth. therefore, some countries adduce free higher education, and this provide more skilled labor which will increase the gross domestic product and per capita income that will affect positively on the purchasing power in the country. skilled labor in the different areas such as technology, health, education, and management will enhance the economic growth, that will improve the welfare in different aspects; social, health and economic.

Adam Smith mentioned the concept of human capital as knowledge, training, talent and experience in 18th century and he explain that the education and well training increase the wealth of society. Schultz (1961) stated there is a variation in workers' wages because of differentials in education and health level as components of the human capital. Schultz imputed the variation of national output between countries to the level of investment on human capital. Becker (1962) suggested that the investment in education and training for the labor in any organization will increase their abilities and improve the production and its quality. Liu and Wong (1981) explained that the determination of the main sectors for the investment will cumulated the physical capital; one of these sectors is the human resources which is one of the basic assets to achieve efficiency economic growth. the investment in human resources will improve the skills, the abilities and upgrade the human capital. Lucas (1988) presented that schooling can transform the unskilled labors into a proficient labor. Schooling is one of the main factors to develop

the human capital that will grow the per capita productivity sustainably. Benhabib and Spiegel (1994) estimated a Cobb-Douglas production function using data for the physical and human capital stocks as explanatory variables. but the human capital was insignificant variable in Cobb-Douglas production function. therefore, they estimated a model consist of the growth productivity rate and human capital stock. they found that the growth productivity rate increase as the human capital stock increased. Fogel (1994) discussed that the development in public health, medical care, and the technological change between 1790 and 1980 in British contributed to 30% of the income growth. Temple (1999) shows that the investment in human capital and its stock construes the variations between countries in per capita income growth. Which agree with Barro and Lee (1996), and Sachs and Warner (1997) results. Bhargava, Jamison, Lau, and Murray (2001) argue that the adult survival rate as a health indicator has an impact on GDP growth rate. His results revealed a positive effect of adult's survival rate on GDP growth rate in low-income countries. Gyimal-Bermpong and Wilson (2004) investigated the effect of health and human capital on the growth of per capita income. They found that the health stock has a direct positive effect on per capita income level, and the additional health investment in Sub Sahara Africa has positive effect on the economic growth at the macro level. Armstrong (2006) defined that human capital is all capabilities of a human including acquired attributes or innate, whose value could be improved by appropriate investments. Almendarez (2013) suggested that the good education with high quality will develop the economic growth.

The economics and growth studies explain that the improvement in health, human and physical capital continuously will increase the available resources productivity and leads to the technical change, which will enhance the per capita income growth, and the surplus will increase the investments and give further growth in income. Health economics studies revealed that expenditure on health care has a positive effect on income. As Schultz (1999) has explained that nations' health level will determine and ensure the development and economic growth.

Data:

The first set Z : (Z_1, Z_2, Z_3, Z_4) are the dependent variables, which are the human capital indices. Z_1 is defined as the probability of survival to age 5 in a country. Z_2 is measured as the rate of adult survival in a country. Z_1 and Z_2 are the best indicators of the stock of a nation's health human capital (Shultz (1999)). While Z_3 and Z_4 measure the expected level of education and grades; where Z_3 is the Harmonized Test Scores (HTS) as a measure of education quality in a country, and Z_4 is the expected year of school.

Kraay (2018) denoted in his HCI definition to the high-quality education and good health for children today will increase the productivity in future.

The second set W : (W_1, W_2, W_3) are the predictor variables, which are the socio-economic indices. W_1 is the life expectancy at birth stands out as health stock indicator, which denotes to the population wellbeing in a country. W_2 is defined as the annual GDP per capita. W_3 is the government expenditure on education.

Canonical Correlations Analysis:

Canonical Correlation Analysis (CCA) is a multivariate method. Knapp (1978) show that CCA could conduct the univariate and multivariate classic methods. CCA explain the simultaneous correlation between two sets of predicted and dependent variables. Thompson (1991).

This research aims to achieve two goals, which are pursued in canonical correlation analysis. These goals can be introduced as:

1. Looking for the first pair of linear combinations "first canonical variable", one from the human capital index and the other from the socioeconomic index. the first canonical correlation is the maximum correlation between the canonical variables. the coefficients of the canonical variables are known as "canonical weights" which interpret the association between the two sets.
2. Calculate the 'structure correlations' which is the correlation between the canonical variables and the original variables.

The Canonical Correlation Analysis method generate the linear combination for two sets of variables to maximize the correlation.

If $W = (W_1, \dots, W_p)'$ and $Z = (Z_1, \dots, Z_q)'$ are two random vectors, the mean of these vectors are μ_W and μ_Z respectively have covariance matrices Σ_W and Σ_Z .

The combination matrix for the two vectors W and Z is $K' = (W', Z')$ and the K matrix is normally distributed; $K \sim (\mu, \Sigma)$ where:

$\mu' = (\mu'_W, \mu'_Z)$ and

$$\Sigma = \begin{pmatrix} \Sigma_{WW} & \Sigma_{WZ} \\ \Sigma_{ZW} & \Sigma_{ZZ} \end{pmatrix}$$

Where the covariance matrix of W & Z is:

$$\Sigma_{WZ} = E[(X - \mu_W)(Y - \mu_Z)']$$

So, the new variables X and Y are the linear combinations of W & Z

$$X = a' W \quad Y = b' Z$$

For the sample:

Where the mean and covariance of the sample is:

$$S = \begin{pmatrix} S_{WW} & S_{WZ} \\ S_{ZW} & S_{ZZ} \end{pmatrix}$$

Where:

$$\bar{W} = \left(\frac{1}{n} \right) \sum_{i=1}^n W_i \quad \text{and} \quad \bar{Z} = \left(\frac{1}{n} \right) \sum_{i=1}^n Z_i$$

$$\bar{W} = \frac{1}{n} \sum_{i=1}^n W_i$$

$$S_{WW} = \frac{1}{n-1} \sum_{i=1}^n (W_i - \bar{W})(W_i - \bar{W})'$$

$$S_{ZZ} = \frac{1}{n-1} \sum_{i=1}^n (Z_i - \bar{Z})(Z_i - \bar{Z})'$$

$$S_{WZ} = S'_{ZW} = \frac{1}{n-1} \sum_{i=1}^n (W_i - \bar{W})(Z_i - \bar{Z})'$$

Canonical Variates and Correlations:

The new variables X&Y are the canonical variates $X = \alpha' W$ and $Y = \beta' Z$, the sample has these properties:

$$\hat{V}ar(X) = \alpha' S_W \alpha$$

$$\hat{V}ar(Y) = \beta' S_Z \beta$$

$$\hat{C}ov(X, Y) = \alpha' S_{WZ} \beta$$

(X_1, Y_1) is the sample first pair canonical variates with linear combination vectors $\{\alpha_1, \beta_1\}$ that maximize the correlation between the canonical variates:

$$\begin{aligned} \hat{C}orr(X, Y) &= \frac{\hat{C}ov(X, Y)}{\sqrt{\hat{V}ar(X)} \sqrt{\hat{V}ar(Y)}} \\ &= \frac{\alpha' S_{WZ} \beta}{\sqrt{\alpha' S_W \alpha} \sqrt{\beta' S_Z \beta}} \end{aligned} \quad (8)$$

Subject to:

- The first pair canonical variates X_1 and Y_1 have a unit vector that give us the maximum correlation in equation (8).
- The L^{th} pair (X_k, Y_k) have a unit variance matrix that give us the maximum correlation in equation (8) between all options which are uncorrelated with the previous $L-1$ canonical variables pairs.

The Calculation of the Canonical Variates and Correlation:

The L^{th} pair of canonical varieties estimated by:

$$\hat{X} = \hat{\alpha}' S^{-1/2} W \quad \text{and} \quad \hat{Y} = \hat{\beta}' S^{-1/2} Z$$

$\underbrace{\quad}_{L} \quad \underbrace{\quad}_{L} \quad \underbrace{\quad}_{L}$

Where:

- $S^{-1/2} S^{-1} S^{-1/2}$ is the L^{th} eigenvector \hat{X} .
- $S^{-1/2} S^{-1} S^{-1/2}$ is the L^{th} eigenvector \hat{Y} .

$$Y \quad \quad \quad XY \quad X \quad \quad XY \quad Y \quad \quad \quad L$$

And the estimated L^{th} canonical correlation is:

$$\hat{c}_{rr}(X, Y) = \hat{\rho}$$

Where:

$$\bullet \quad S^{-1/2} S^{-1} S \quad S^{-1/2} \quad \text{or} \quad S^{-1/2} S \quad S^{-1} S \quad S^{-1/2} \quad \text{Give us the}$$

$$X \quad \quad \quad XY \quad XY \quad \quad XY \quad \quad \quad XY \quad X \quad XY \quad \quad Y$$

L^{th} eigenvalue $\hat{\rho}$.

- $\hat{X} = \hat{\alpha}W$ where $\hat{\alpha} = [\hat{\alpha}_1, \hat{\alpha}_2, \dots, \hat{\alpha}_p]$ and the p canonical variates in W are $\hat{X} = (\hat{X}_1, \dots, \hat{X}_p)$.
- $\hat{Y} = \hat{\beta}Z$ where $\hat{\beta} = [\hat{\beta}_1, \hat{\beta}_2, \dots, \hat{\beta}_q]$ and the q canonical variates in Z are $\hat{Y} = (\hat{Y}_1, \dots, \hat{Y}_q)$.
- We are used the first p canonical variates in Z if $p \leq q$.

Redundancy Index:

Redundancy Index is the percentage of variance in the actual variables in the first set in the canonical function that explained by the canonical variate in the other set in that function. Dattalo (2014).

Structure coefficients:

Measure the simple linear correlation between the canonical variates and its independent variables. Wickramasinghe, (2019).

Results:

In this research, we seek to quantify the association between human capital and some of the socioeconomic indices. CCA is a suitable technique to determine the correlation and its power among the two sets of the indices. firstly, find the pair linear combinations that maximize the correlation. secondly, we determine the pair of linear combinations with maximum correlation between other pairs which are uncorrelated with the initially selected pair and so on.

Using the World Bank data (2018) for 108 countries; we measured the canonical correlation between the human capital indices: probability of survival to age 5, adult survival rate, expected years of school, and harmonized test scores as dependent variables, and socioeconomic indices: life expectancy at birth, GDP per capita and government expenditure on education as predictor variables.

Copulatively, the full model across all functions was statistically significant using the Wilks's $\Lambda = 0.0243$ criterion, $F = 68.593$, and p -value = 0.000. Wilks's Λ is the unexplained variance by this model, $1 - \Lambda$ yields the explained variance of the full model. Therefore, the full model can explain a substantial portion, about 97.6% of the variance shared between the human capital and socio-economic indices.

The hierarchical arrangement of functions could be tested by the Dimension Reduction Analysis. As the result show above, the functions (1-3) were highly significant since the criterion of Wilks's $\Lambda = 0.0243$, and the F - statistic =

68.593 with p-value = 0.000. And the Functions 2–3 were also statistically significant with criterion of Wilks’s $\Lambda = 0.697$, and the F -statistic = 6.72 with p-value = 0.000. The test of Function 3 was also statistically significant as the criterion of Wilks’s Λ show that its value is 0.899 with F- statistic = 5.767, and its p-value = 0.003.

Table (1): Canonical Variates – Correlation and Redundancy Index

Canonical variate	Canonical Correlation	Square Canonical Correlation	Redundancy Index for W variables	Redundancy Index for Z variables
1	0.98	0.9604	0.7395	0.4705
2	0.47	0.2209	0.0026	0.0375
3	0.32	0.1024	0.0012	0.0348

CCA is used to investigate the relation between human capital and socio- economic indices. Table (1) show the correlation of each successive function was 0.98, 0.47, and 0.32. However, the redundancy index of all functions except the first one is about zero. Therefore, the first canonical correlation is noteworthy in the context of this study.

The first redundancy index for the predictor's variables (W) is 74%, which means that the first canonical variate of socio-economic indices explains 74% of the human capital indices in the model. Analogously, the first canonical variate of human capital indices explains 47% of the socio- economic indices in the model as the first Redundancy Index for the dependent's variables (Z) show. Each Redundancy Index denote that the canonical variates for the socio – economic indices explain more variation in human capital indices in the first function than the canonical variate for the human capital indices explained in the set of socio – economic indices. Dattalo (2014).

The first canonical variate pair is:

$$X_1 = 0.135 W_1 + 0.0001 W_2 - 0.009 W_3$$

$$Y_1 = 10.836 Z_1 + 6.46 Z_2 + 0.001 Z_3 + 0.0168 Z_4$$

The first canonical variate (standardized coefficients) pair is:

$$X_1 = 1.06 (W_1 - \bar{W})/S_{W1} + (-0.09) (W_2 - \bar{W})/S_{W2} + (-0.02) (W_3 - \bar{W})/S_{W3}$$

$$Y_1 = 0.34 (Z_1 - \bar{Z})/S_{Z1} + 0.04 (Z_2 - \bar{Z})/S_{Z2} + 0.07 (Z_3 - \bar{Z})/S_{Z3} + 0.60 (Z_4 - \bar{Z})/S_{Z4}$$

Table (2): Structure coefficients (Canonical Loadings)

W's variables	canonical variates: X ₁	Z's variables	canonical variates: Y ₁
Life expectancy at birth	0.99763	Probability of Survival to Age 5	0.94518
GDP per capita	0.64951	Adult Survival Rate	0.84681
Government expenditure on education	0.20253	Harmonized Test Scores	0.79468

		Expected Years of School	0.97260
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The first canonical variate X_1 can be interpreted as health and economic status with positive weight in this variate and education expenditure with negative coefficient. First canonical variate Y_1 have positive weights with all the dependent variables in the model, which represent health and education human capital stock in a country. The first pair of canonical variates are correlated strongly. The first canonical variates explain 96.5% of the variance shared between the human capital and socio-economic indices. Cooley and Lohnes (1971).

Since the life expectancy at birth is correlated very strongly and positively with the first canonical variate U_1 , the life expectancy at birth contributed more to the first canonical variate U_1 than GDP per capita and government expenditure on education. GDP per capita has a moderate and positive correlation with the first canonical variate U_1 , and the first canonical variate is correlated weakly with percentage of government expenditure on education. The first canonical variable U_1 explain 49% of the variance in predictor variables. Table (2).

The first canonical variate V_1 is correlated strongly and positively with all dependent variables in the model. But expected years of school and probability of survival to age 5 contributed more to the first canonical variate V_1 than the adult survival rate and Harmonized Test Scores (HTS) relatively. The first canonical variate V_1 explain 77% of the variance in dependent variables. Table (2).

Discussion:

This study investigates the association between human capital indices and socio – economic indices using CCA. Instead of using separated linear regression models for each human capital indicator, we used the CCA in view the fact that it measures the effects of multiple predictor variables on multiple dependent variables simultaneously in one model. Since the CCA invests all the information in the two groups of variables and arrives at an efficient estimate of the relationship between the two groups, the CCA presents a more efficient model for measuring the effects of socio- economic indicators on human capital indicators. CCA used the two sets predictor and dependent set to limit the inefficiencies multiple testing that may occur to reduce type one error. Furthermore, the latent variable method averts the multicollinearity problem to arise, Liu, Drane, and Wu (2009). The result of CCA gives a global view of the association between indicators of human capital and socio-economic. We found that 97.6% of the variance shared between the human capital and socio-economic indices. In addition to providing an assessment of the association between two sets of variables, the application of CCA helped in determining the interaction between the two sets that might contribute to the relationship based on the maximum conical correlation of the first pair of canonical variates, which explains 96.5% of the variance shared between the human capital and socio-economic indices. In addition to, the first redundancy index for socio-economic indices explains 74% of the human capital indices in the model and the human capital indices explains 47% of the socio-economic indices in the model. As Sherry and Henson (2005), structure coefficients are answers to the question “what variables are contributing to the relationship between the variables set across the functions?”. The life expectancy at birth has got the highest structure coefficient, which is a common and important predictor for human capital progress. The second variable among the predictors sets (based on its structure coefficient) is GDP per capita. Thus, the country with high GDP per capita has a positive and significant effect on the human capital. The health economics literature indicates a positive effect of income on expenditure on health care Gyimal- Bermpong and Wilson (2004). In this study, the variable of government expenditure on education got the third place among the predictors. The expected years of school and probability of survival to age 5 have got higher structure coefficient than adult survival rate and Harmonized Test Scores (HTS) relatively, which are common and important for the socio – economic indices development. The expected years of school and Harmonized Test Scores (HTS) reveal the quality of education in a country which are the key to the development of human capital Lucas (1988), and economic growth (Wong, (1981) and Almendarez (2013)). Variation of human capital among different countries and its investment explain the variation between countries in economic growth (Barro and Lee (1996), Sachs and Warner (1997), and Temple (1999)). Probability of survival to age 5 and adult survival rate determine the health capital stock level in a country; they have a positive and direct impact on GDP per capita growth.

Conclusion:

Inequality in human capital between countries refer to socio – economic indicators variation. The main socio-economic indicator associated with human capital indices is the life expectancy at birth. Life expectancy at birth is the main cumulative indicator for the health capital stock in the country. Life expectancy at birth and GDP per capita have an effective contribution to the human capital indices.

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