Journal for Re Attach Therapy and Developmental Diversities
eISSN: 2589-7799
2023 November; 6(10s)2: 2133-2137

Intuitionistic Fuzzy $\widehat{g}^*$ Semi Connectedness in Intuitionistic Fuzzy Topological Spaces

S. Pious Missier[1], A. Peter Arokiaraj[2*], A.Nagarajan[3]

[1]Head & Associate Professor (Rtd.), Department of Mathematics, Don Bosco College of Arts and Science, (Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627 012), Keela Eral, Thoothukudi, Tamil Nadu-628 908, India, ***spmissier@gmail.com***
[2*]Research Scholar (Reg.No-19222232091032), PG and Research Department of Mathematics, V.O.Chidambaram College, (Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627 012), Thoothukudi-628 008, India, ***petrajsdb@gmail.com***
[3]Head & Associate Professor (Rtd.), PG and Research Department of Mathematics, V.O.Chidambaram College, (Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627 012), Thoothukudi-628 008, India, ***nagarajan.voc@gmail.com***

Abstract:

This article is envisioned to navigate ***Intuitionistic Fuzzy $\widehat{g}^*$Semi Connectedness*** in Intuitionistic fuzzy topological spaces. An investigative study on Intuitionistic Fuzzy $\widehat{g}^*$Semi connected spaces, Intuitionistic Fuzzy $\widehat{g}^*$Semi super connected spaces and Intuitionistic Fuzzy $\widehat{g}^*$Semi extremely disconnected spaces is elucidated with proper explanations and examples.

Key Words: Intuitionistic Fuzzy $\widehat{g}^*$ Semi Closed set($\mathcal{IF}\widehat{g}^*s\mathcal{C}$), Intuitionistic Fuzzy $\widehat{g}^*$ Semi Connected Spaces($\mathcal{IF}\widehat{g}^*sConS$) and Intuitionistic Fuzzy $\widehat{g}^*$ Semi Super Connected Spaces($\mathcal{IF}\widehat{g}^*sSConS$) and Intuitionistic Fuzzy $\widehat{g}^*$ Semi extremally disconnected Spaces($\mathcal{IF}\widehat{g}^*sEDConS$)

AMS Subject Classifications (2000): 54A40, 03F55

I. INTRODUCTION

Zadeh[8] in 1965 opened a new page in Mathematics with fuzzy sets to deal with ambiguity. Chang(1968)[2] established it into fuzzy topology. Atanassov[1] with his cognitive idea brought forth intuitionistic fuzzy set and later Coker[3] generalized it into intuitionistic fuzzy topological spaces. Pious Misser et al.,[5] recently illustrated Intuitionistic Fuzzy $\widehat{g}^*$Semi Closed sets. Taking a lead from there, we proceed ahead to explore Intuitionistic Fuzzy $\widehat{g}^*$Semi connected spaces, Intuitionistic Fuzzy $\widehat{g}^*$Semi super connected spaces and Intuitionistic Fuzzy $\widehat{g}^*$Semi extremally disconnected spaces in Intuitionistic fuzzy topological spaces.

II. PRELIMINARIES

Definition 2.1. [1] Let $\mathbb{U}$ be a universal set. Then $\mathbb{G}_{if} = \{\langle \mathbb{u}, \mu_{\mathbb{G}_{if}}(\mathbb{u}), \upsilon_{\mathbb{G}_{if}}(\mathbb{u})\rangle : \mathbb{u} \in \mathbb{U}\}$ is called as an intuitionistic fuzzy subset ($\mathcal{IFS}$ in short) in $\mathbb{U}$. Here the functions $\mu_{\mathbb{G}_{if}}: \mathbb{U} \to [0,1]$ and $\upsilon_{\mathbb{G}_{if}}: \mathbb{U} \to [0,1]$ denote the degree of membership (namely $\mu_{\mathbb{G}_{if}}(\mathbb{u})$) and the degree of non-membership (namely $\upsilon_{\mathbb{G}_{if}}(\mathbb{u})$) of each element $\mathbb{u} \in \mathbb{U}$ to the set $\mathbb{G}_{if}$ respectively, and $0 \leq \mu_{\mathbb{G}_{if}}(\mathbb{u}) + \upsilon_{\mathbb{G}_{if}}(\mathbb{u}) \leq 1$ for each $\mathbb{u} \in \mathbb{U}$. The set of all intuitionistic fuzzy sets in $\mathbb{U}$ is denoted by $\mathcal{IFS}$s($\mathbb{U}$). For any two $\mathcal{IFS}$s $\mathbb{G}_{if}$ and $\mathbb{H}_{if}$, $(\mathbb{G}_{if} \cup \mathbb{H}_{if})^C = \mathbb{G}_{if}^C \cap \mathbb{H}_{if}^C$; $(\mathbb{G}_{if} \cap \mathbb{H}_{if})^C = \mathbb{G}_{if}^C \cup \mathbb{H}_{if}^C$.

Definition2.2: [1] If $\mathbb{G}_{if} = \{\langle \mathbb{u}, (\mathbb{u}), \upsilon_{\mathbb{G}_{if}}(\mathbb{u})\rangle : \mathbb{u} \in \mathbb{U}\}$ and $\mathbb{H}_{if} = \{\langle \mathbb{u}, \mu_{\mathbb{H}_{if}}(\mathbb{u}), \upsilon_{\mathbb{H}_{if}}(\mathbb{u})\rangle : \mathbb{u} \in \mathbb{U}\}$ be two $\mathcal{IFS}s(\mathbb{U})$, then
(a) $\mathbb{G}_{if} \subseteq \mathbb{H}_{if}$ if and only if $\mu_{\mathbb{G}_{if}} \leq \mu_{\mathbb{H}_{if}}$ and $\upsilon_{\mathbb{G}_{if}}(\mathbb{u}) \geq \upsilon_{\mathbb{H}_{if}}(\mathbb{u})$ for all $x \in \mathbb{X}$,
(b) $\mathbb{G}_{if} = \mathbb{H}_{if}$ if and only if $\mathbb{G}_{if} \subseteq \mathbb{H}_{if}$ and $\mathbb{G}_{if} \supseteq \mathbb{H}_{if}$,
(c) $\mathbb{G}_{if}^C = \{\langle \mathbb{u}, \upsilon_{\mathbb{G}_{if}}(\mathbb{u}), \mu_{\mathbb{G}_{if}}(\mathbb{u})\rangle : \mathbb{u} \in \mathbb{U}\}$ (complement of $\mathbb{G}_{if}$),
(d) $\mathbb{G}_{if} \cup \mathbb{H}_{if} = \{\langle \mathbb{u}, \mu_{\mathbb{G}_{if}}(\mathbb{u}) \vee \mu_{\mathbb{H}_{if}}(x), \upsilon_{\mathbb{G}_{if}}(\mathbb{u}) \wedge \upsilon_{\mathbb{H}_{if}}(\mathbb{u})\rangle : \mathbb{u} \in \mathbb{U}\}$,
(e) $\mathbb{G}_{if} \cap \mathbb{H}_{if} = \{\langle \mathbb{u}, \mu_{\mathbb{G}_{if}}(\mathbb{u}) \wedge \mu_{\mathbb{H}_{if}}(x), \upsilon_{\mathbb{G}_{if}}(\mathbb{u}) \vee \upsilon_{\mathbb{H}_{if}}(\mathbb{u})\rangle : \mathbb{u} \in \mathbb{U}\}$,
(f) $(\mathbb{G}_{if} \cup \mathbb{H}_{if})^C = \mathbb{G}_{if}^C \cap \mathbb{H}_{if}^C$ and $(\mathbb{G}_{if} \cap \mathbb{H}_{if})^C = \mathbb{G}_{if}^C \cup \mathbb{H}_{if}^C$.
(h) $\tilde{\mathbf{0}} = \langle \mathbb{u}, 0, 1\rangle$(empty set) and $\tilde{\mathbf{1}} = \langle \mathbb{u}, 1, 0\rangle$ (whole set).

Definition 2.3. [3] An intuitionistic fuzzy topology ($\mathcal{IFT}$) on $\mathbb{U}$ is a family of $\mathcal{IFS}s$ in $\mathbb{U}$, satisfying the following axioms.
1. $\tilde{0}$, $\tilde{1} \in \tau_{if}$
2. $\mathbb{G}_{if} \cap \mathbb{H}_{if} \in \tau_{if}$ for any $\mathbb{G}_{if}$, $\mathbb{H}_{if} \in \tau_{if}$
3. $\cup \mathbb{G}_{{if}_i} \in \tau_{if}$ for any family $\{\mathbb{G}_{{if}_i} / i \in \mathcal{J}\} \subseteq \tau_{if}$.

The pair $(\mathbb{U},\tau_{if})$ is called an intuitionistic fuzzy topological space ($\mathcal{IFTS}$) and any $\mathcal{IFS}$ in τ_{if} is known as an intuitionistic fuzzy open set ($\mathcal{IFOS}$) in $\mathbb{X}$. The complement ($\mathbb{G}_{if}^{C}$) of an $\mathcal{IFOS}$ $\mathbb{G}_{if}$ in an $\mathcal{IFTS}(\mathbb{U}, \tau_{if})$ is called an intuitionistic fuzzy closed set($\mathcal{IFCS}$) in $\mathbb{U}$. In this paper, Intuitionistic fuzzy interior is denoted by int_{if} and Intuitionistic fuzzy closure is denoted by cl_{if}.

Definition 2.4. [3] Let $(\mathbb{U},\tau_{if})$ be an $\mathcal{IFTS}$ and $\mathbb{G}_{if} = \{\langle \mathbb{u} , \mu_{\mathbb{G}_{if}}(\mathbb{u}), \upsilon_{\mathbb{G}_{if}}(\mathbb{u})\rangle : \mathbb{u} \in \mathbb{U} \}$ be an $\mathcal{IFS}$ in $\mathbb{X}$. Then the interior and closure of the above $\mathcal{IFS}$ are defined as follows:
(i) $int_{if}(\mathbb{G}_{if}) = \cup \{\mathcal{H}_{if} \mid \mathcal{H}_{if}$ is an $\mathcal{IFOS}$ in $\mathbb{X}$ and $\mathcal{H}_{if} \subseteq \mathbb{G}_{if}\}$
(ii) $cl_{if}(\mathbb{G}_{if}) = \cap \{\mathcal{K}_{if} \mid \mathcal{K}_{if}$ is an $\mathcal{IFCS}$ in $\mathbb{X}$ and $\mathbb{G}_{if} \subseteq \mathcal{K}_{if}\}$
(iii) $cl_{if}(\mathbb{G}_{if}{}^{c}) = (int_{if}(\mathbb{G}_{if}))^{c}$
(iv) $int_{if}(\mathbb{G}_{if}{}^{c}) = (cl_{if}(\mathbb{G}_{if}))^{c}$

Definition 2.5. [5] An $\mathcal{IFS}$ $\mathbb{G}_{if}$ of an $\mathcal{IFTS}$ $(\mathbb{U}, \tau_{if})$ is called an $\mathcal{IF}\hat{g}^{*}s\mathcal{CS}$, if $scl_{if}(\mathbb{G}_{if}) \subseteq \mathcal{O}_{if}$ whenever $\mathbb{G}_{if} \subseteq \mathcal{O}_{if}$ and $\mathcal{O}_{if}$ is any $\mathcal{IF}\hat{g}\mathcal{O}$ in $(\mathbb{U}, \tau_{if})$.

Definition 2.4. [5] Let $(\mathbb{U},\tau_{if})$ be an $\mathcal{IFTS}$ and $\mathbb{G}_{if} = \{\langle \mathbb{u}, \mu_{\mathbb{G}_{if}}(\mathbb{u}), \upsilon_{\mathbb{G}_{if}}(\mathbb{u})\rangle : \mathbb{u} \in \mathbb{U} \}$ be an $\mathcal{IFS}$ in $\mathbb{U}$. Then
(i) $\hat{g}^{*}sint_{if}(\mathbb{G}_{if}) = \bigcup\{\mathcal{H}_{if} \mid \mathcal{H}_{if}$ is an $\mathcal{IF}\hat{g}^{*}s\mathcal{OS}$ in $\mathbb{U}$ and $\mathcal{H}_{if} \subseteq \mathbb{G}_{if}\}$,
(ii) $\hat{g}^{*}scl_{if}(\mathbb{G}_{if}) = \bigcap\{\mathcal{K}_{if} \mid \mathcal{K}_{if}$ is an $\mathcal{IF}\hat{g}^{*}s\mathcal{CS}$ in $\mathbb{U}$ and $\mathbb{G}_{if} \subseteq \mathcal{K}_{if}\}$
(iii) $\hat{g}^{*}scl_{if}(\mathbb{G}_{if}{}^{c}) = (\hat{g}^{*}sint_{if}(\mathbb{G}_{if}))^{c}$
(iv) $\hat{g}^{*}sint_{if}(\mathbb{G}_{if}{}^{c}) = (\hat{g}^{*}scl_{if}(\mathbb{G}_{if}))^{c}$

Definition 2.5. [5] An $\mathcal{IFTS}$ $(\mathbb{U},\tau_{if})$ is called an $\mathcal{IF}$ $\hat{g}^{*}$ semi $T^{*}_{1/2}$ space ($\mathcal{IF}\hat{g}^{*}s$ $T^{*}_{1/2}$ space) if every $\mathcal{IF}\hat{g}^{*}s\mathcal{CS}$ is $\mathcal{IFCS}$ in $(\mathbb{U},\tau_{if})$.

Definition 2.6.[6] A mapping $f: (\mathbb{U}, \tau_{if}) \rightarrow (\mathbb{V}, \sigma_{if})$ is called an $\mathcal{IF}\hat{g}^{*}s$ -continuous if $f^{-1}(\mathbb{G}_{if})$ is an $\mathcal{IF}\hat{g}^{*}s\mathcal{C}$ set in $(\mathbb{U},\tau_{if})$ for every $\mathcal{IFC}$ set $\mathbb{G}_{if}$ of $(\mathbb{V}, \sigma_{if})$.

Definition 2.7. [6] A mapping $f: (\mathbb{U}, \tau_{if}) \rightarrow (\mathbb{V}, \sigma_{if})$ is called an $\mathcal{IF}\hat{g}^{*}s$ -irresolute if $f^{-1}(\mathbb{G}_{if})$ is an $\mathcal{IF}\hat{g}^{*}s\mathcal{C}$ set in $(\mathbb{U},\tau_{if})$ for every $\mathcal{IF}\hat{g}^{*}s\mathcal{C}$ set $\mathbb{G}_{if}$ of $(\mathbb{V}, \sigma_{if})$.

Definition 2.8. [7] Two $\mathcal{IFS}$s $\mathbb{G}_{if}$ and $\mathbb{H}_{if}$ are said to be *q-coincident* ($\mathbb{G}_{if\ q}$ $\mathbb{H}_{if}$ in short) iff there exists an element $\mathbb{u} \in \mathbb{U}$ such that $\mu_{\mathbb{G}_{if}}(\mathbb{u}) > \upsilon_{\mathbb{H}_{if}}(\mathbb{u})$ or $\upsilon_{\mathbb{G}_{if}}(\mathbb{u}) < \mu_{\mathbb{H}_{if}}(\mathbb{u})$.

Definition 2.9. [7] Two $\mathcal{IFS}$s $\mathbb{G}_{if}$ and $\mathbb{H}_{if}$ are said to be *not q-coincident* ($\mathbb{G}_{if\ q^{c}}$ $\mathbb{H}_{if}$ in short) iff $\mathbb{G}_{if} \subseteq \mathbb{H}_{if}{}^{c}$.

Definition 2.10. [7] An $\mathcal{IFTS}$ $(\mathbb{U},\tau_{if})$ is called *intuitionistic fuzzy C_5-connected*($\mathcal{IFC}_5ConS$) if the only $\mathcal{IFS}$s which are both $\mathcal{IFO}$ and $\mathcal{IFC}$ are $\tilde{\mathbf{0}}$ and $\tilde{\mathbf{1}}$.

III. INTUITIONISTIC FUZZY $\hat{g}^{*}$ SEMI CONNECTED SPACES

Definition 3.1. An $\mathcal{IFTS}$ $(\mathbb{U},\tau_{if})$ is called $\mathcal{IF}$ $\hat{g}^{*}$ Semi Connected Space($\mathcal{IF}\hat{g}^{*}sConS$) if the only $\mathcal{IFS}$s which are both $\mathcal{IF}\hat{g}^{*}s\mathcal{CS}$ and $\mathcal{IF}\hat{g}^{*}s\mathcal{OS}$ are $\tilde{\mathbf{0}}$ and $\tilde{\mathbf{1}}$.

Theorem 3.2. Every $\mathcal{IF}\hat{g}^{*}sConS$ is $\mathcal{IFC}_5ConS$ but not conversely.

Proof: Let $(\mathbb{U},\tau_{if})$ is called $\mathcal{IF}\hat{g}^{*}sConS$. Let us presume $(\mathbb{U},\tau_{if})$ is not an $\mathcal{IFC}_5ConS$, then there exists an $\mathcal{IFS}$ $\mathbb{G}_{if}$ that is both $\mathcal{IFCS}$ and $\mathcal{IFOS}$ in $(\mathbb{U},\tau_{if})$. That is, $\mathbb{G}_{if}$ is both $\mathcal{IF}\hat{g}^{*}s\mathcal{CS}$ and $\mathcal{IF}\hat{g}^{*}s\mathcal{OS}$ in $(\mathbb{U},\tau_{if})$. This implies that $(\mathbb{U},\tau_{if})$ is not an $\mathcal{IF}\hat{g}^{*}sConS$. This contradicts our assumption. Therefore $(\mathbb{U},\tau_{if})$ must be an $\mathcal{IFC}_5ConS$.

Example 3.3. Let $\mathbb{U} = \{\mathfrak{e}, \mathfrak{f}\}$, $\tau_{\mathrm{if}} = \{\tilde{0}, \mathbb{G}_{\mathrm{if}}, \tilde{1}\}$ where $\mathbb{G}_{\mathrm{if}}$={<$\mathfrak{e}$, 0.3, 0.7>, <$\mathfrak{f}$, 0.4, 0.6>}. Then $(\mathbb{U},\tau_{\mathrm{if}})$ is $\mathcal{IFC}_5ConS$, but not $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}ConS$, because the $\mathcal{IFS}$ $\mathbb{N}_{\mathrm{if}}$ ={<$\mathfrak{e}$, 0.4, 0.6>, <$\mathfrak{f}$, 0.5, 0.5>} in $(\mathbb{U},\tau_{\mathrm{if}})$ is both $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{CS}$ and $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OS}$.

Theorem 3.4. An $\mathcal{IFTS}$ $(\mathbb{U},\tau_{\mathrm{if}})$ is an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}ConS$ iff there exist no non-zero $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OS}$s $\mathbb{G}_{\mathrm{if}}$ and $\mathbb{H}_{\mathrm{if}}$ in $(\mathbb{U},\tau_{\mathrm{if}})$ such that $\mathbb{H}_{\mathrm{if}}= \mathbb{G}_{\mathrm{if}}{}^{c}$, $\mathbb{H}_{\mathrm{if}}= (\mathscr{s}cl_{\mathrm{if}}(\mathbb{G}_{\mathrm{if}}))^c$ and $\mathbb{G}_{\mathrm{if}}= (\mathscr{s}cl_{\mathrm{if}}(\mathbb{H}_{\mathrm{if}}))^c$.

Proof: ***Necessity:*** Let $(\mathbb{U},\tau_{\mathrm{if}})$ be an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}ConS$ We assume two $\mathcal{IFS}$s $\mathbb{G}_{\mathrm{if}}$ and $\mathbb{H}_{\mathrm{if}}$ such that $\mathbb{G}_{\mathrm{if}} \neq \tilde{0} \neq \mathbb{H}_{\mathrm{if}}$, $\mathbb{H}_{\mathrm{if}}=\mathbb{G}_{\mathrm{if}}{}^{c}$, $\mathbb{H}_{\mathrm{if}}=(\mathscr{s}cl_{\mathrm{if}}(\mathbb{G}_{\mathrm{if}}))^c$ and $\mathbb{G}_{\mathrm{if}}=(\mathscr{s}cl_{\mathrm{if}}(\mathbb{H}_{\mathrm{if}}))^c$. Since $(\mathscr{s}cl_{\mathrm{if}}(\mathbb{G}_{\mathrm{if}}))^c$ and $(\mathscr{s}cl_{\mathrm{if}}(\mathbb{H}_{\mathrm{if}}))^c$ are $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OS}$s in $(\mathbb{U},\tau_{\mathrm{if}})$, $\mathbb{G}_{\mathrm{if}}$ and $\mathbb{H}_{\mathrm{if}}$ are $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OS}$s in $(\mathbb{U},\tau_{\mathrm{if}})$. This implies $(\mathbb{U},\tau_{\mathrm{if}})$ is not an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}ConS$, which contradicts our assumption. Therefore there exist no non-zero $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OS}$s $\mathbb{G}_{\mathrm{if}}$ and $\mathbb{H}_{\mathrm{if}}$ in $(\mathbb{U},\tau_{\mathrm{if}})$ such that $\mathbb{H}_{\mathrm{if}}=\mathbb{G}_{\mathrm{if}}{}^{c}$, $\mathbb{H}_{\mathrm{if}}=(\mathscr{s}cl_{\mathrm{if}}(\mathbb{G}_{\mathrm{if}}))^c$ and $\mathbb{G}_{\mathrm{if}}= (\mathscr{s}cl_{\mathrm{if}}(\mathbb{H}_{\mathrm{if}}))^c$.

Sufficiency: Let $\mathbb{G}_{\mathrm{if}}$ be both $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OS}$ and $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{CS}$ in $(\mathbb{U},\tau_{\mathrm{if}})$ such that $\tilde{1} \neq \mathbb{G}_{\mathrm{if}} \neq \tilde{0}$. Now by taking $\mathbb{H}_{\mathrm{if}}=\mathbb{G}_{\mathrm{if}}{}^{c}$, we obtain a contradiction to our hypothesis. Hence $(\mathbb{U},\tau_{\mathrm{if}})$ is an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}ConS$.

Theorem 3.5. Let $(\mathbb{U},\tau_{\mathrm{if}})$ be an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}T^*_{1/2}$ space, then the following statements are equivalent: (a) $(\mathbb{U},\tau_{\mathrm{if}})$ is an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}ConS$, (b) $(\mathbb{U},\tau_{\mathrm{if}})$ is an $\mathcal{IFC}_5ConS$.

Proof: (a) ⇒ (b) is evident from Theorem 3.1.

(b) ⇒ (a) Let $(\mathbb{U},\tau_{\mathrm{if}})$ be an $\mathcal{IFC}_5ConS$. Suppose $(\mathbb{U},\tau_{\mathrm{if}})$ is not $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}ConS$, then there is an existence of a proper $\mathcal{IFS}$ $\mathbb{G}_{\mathrm{if}}$ in $(\mathbb{U},\tau_{\mathrm{if}})$ which is both $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{CS}$ and $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OS}$. But since $(\mathbb{U},\tau_{\mathrm{if}})$ is an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}$ $T^*_{1/2}$ space, $\mathbb{G}_{\mathrm{if}}$ in $(\mathbb{U},\tau_{\mathrm{if}})$ is both $\mathcal{IFCS}$ and $\mathcal{IFOS}$. This implies that $(\mathbb{U},\tau_{\mathrm{if}})$ is not $\mathcal{IFC}_5ConS$. This contradicts our assumption. Therefore $(\mathbb{U},\tau_{\mathrm{if}})$ must be an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}ConS$.

Theorem 3.6. If $f: (\mathbb{U}, \tau_{\mathrm{if}}) \to (\mathbb{V}, \sigma_{\mathrm{if}})$ is an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}$–continuous surjective mapping and $(\mathbb{U},\tau_{\mathrm{if}})$ is an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}ConS$, then $(\mathbb{V}, \sigma_{\mathrm{if}})$ is an $\mathcal{IFC}_5ConS$.

Proof: Let $(\mathbb{U},\tau_{\mathrm{if}})$ be an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}ConS$. Suppose $(\mathbb{V}, \sigma_{\mathrm{if}})$ is not $\mathcal{IFC}_5ConS$, then there is an existence of a proper $\mathcal{IFS}$ $\mathbb{G}_{\mathrm{if}}$ in $(\mathbb{V}, \sigma_{\mathrm{if}})$ which is both $\mathcal{IFCS}$ and $\mathcal{IFOS}$. Since f is an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}$ –continuous surjective mapping, $f^{-1}(\mathbb{G}_{\mathrm{if}})$ is both $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{CS}$ and $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OS}$ in $(\mathbb{U},\tau_{\mathrm{if}})$. This contradicts our assumption. Hence $(\mathbb{V}, \sigma_{\mathrm{if}})$ must be an $\mathcal{IFC}_5ConS$.

Theorem 3.7. If $f: (\mathbb{U}, \tau_{\mathrm{if}}) \to (\mathbb{V}, \sigma_{\mathrm{if}})$ is an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}$–irresolute surjective mapping and $(\mathbb{U},\tau_{\mathrm{if}})$ is an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}ConS$, then $(\mathbb{V}, \sigma_{\mathrm{if}})$ is an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}ConS$.

Proof: Let $(\mathbb{U},\tau_{\mathrm{if}})$ be an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}ConS$. Suppose $(\mathbb{V}, \sigma_{\mathrm{if}})$ is not an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}ConS$, then there is an existence of a proper $\mathcal{IFS}$ $\mathbb{G}_{\mathrm{if}}$ in $(\mathbb{V}, \sigma_{\mathrm{if}})$ which is both $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{CS}$ and $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OS}$. Since f is an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}$ – irresolute surjective mapping, $f^{-1}(\mathbb{G}_{\mathrm{if}})$ is both $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{CS}$ and $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OS}$ in $(\mathbb{U},\tau_{\mathrm{if}})$. This contradicts our assumption. Hence $(\mathbb{V}, \sigma_{\mathrm{if}})$ must be an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}ConS$.

Definition 3.8. An $\mathcal{IFTS}$ $(\mathbb{U},\tau_{\mathrm{if}})$ is $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}Con$ between two $\mathcal{IFS}$s $\mathbb{G}_{\mathrm{if}}$ and $\mathbb{H}_{\mathrm{if}}$ if there is no $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OS}$ $\mathbb{I}_{\mathrm{if}}$ in $(\mathbb{U},\tau_{\mathrm{if}})$ such that $\mathbb{G}_{\mathrm{if}} \subseteq \mathbb{I}_{\mathrm{if}}$ and $\mathbb{I}_{\mathrm{if}\ q}{}^{c}\ \mathbb{H}_{\mathrm{if}}$.

Theorem 3.9. If an $\mathcal{IFTS}$ $(\mathbb{U},\tau_{\mathrm{if}})$ is $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}Con$ between two $\mathcal{IFS}$s $\mathbb{G}_{\mathrm{if}}$ and $\mathbb{H}_{\mathrm{if}}$, then it is $\mathcal{IFC}_5Con$ between $\mathbb{G}_{\mathrm{if}}$ and $\mathbb{H}_{\mathrm{if}}$ but the converse need not be true.

Proof: Suppose $(\mathbb{U},\tau_{\ \mathrm{if}})$ is not $\mathcal{IFC}_5Con$ between $\mathbb{G}_{\mathrm{if}}$ and $\mathbb{H}_{\mathrm{if}}$, then there exists and $\mathcal{IFOS}$ $\mathbb{I}_{\mathrm{if}}$ in $(\mathbb{U},\tau_{\mathrm{if}})$ such that $\mathbb{G}_{\mathrm{if}} \subseteq \mathbb{I}_{\mathrm{if}}$ and $\mathbb{I}_{\mathrm{if}\ q}{}^{c}\ \mathbb{H}_{\mathrm{if}}$. Since every $\mathcal{IFOS}$ is an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OS}$, there exists an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OS}$ $\mathbb{I}_{\mathrm{if}}$ such that $\mathbb{G}_{\mathrm{if}} \subseteq \mathbb{I}_{\mathrm{if}}$ and $\mathbb{I}_{\mathrm{if}\ q}{}^{c}\ \mathbb{H}_{\mathrm{if}}$. This implies $(\mathbb{U},\tau_{\mathrm{if}})$ is not $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}Con$ between $\mathbb{G}_{\mathrm{if}}$ and $\mathbb{H}_{\mathrm{if}}$, which contradicts the assumption. Therefore $(\mathbb{U},\tau_{\mathrm{if}})$ must be $\mathcal{IFC}_5Con$ between $\mathbb{G}_{\mathrm{if}}$ and $\mathbb{H}_{\mathrm{if}}$.

Example 3.10. Let $\mathbb{U} = \{\mathfrak{e}, \mathfrak{f}\}$, $\tau_{\mathrm{if}} = \{\tilde{0}, \mathbb{G}_{\mathrm{if}}, \tilde{1}\}$ where $\mathbb{G}_{\mathrm{if}}$={<$\mathfrak{e}$, 0.5, 0.5>, <$\mathfrak{f}$, 0.4, 0.6>}. Let $\mathbb{A}_{\mathrm{if}}$={<$\mathfrak{e}$, 0.52, 0.48>, <$\mathfrak{f}$, 0.43, 0.57>} and $\mathbb{B}_{\mathrm{if}}$={<$\mathfrak{e}$, 0.6, 0.4>, <$\mathfrak{f}$, 0.7, 0.3>} be two $\mathcal{IFS}$s in $\mathbb{U}$. Then $(\mathbb{U},\tau_{\mathrm{if}})$ is $\mathcal{IFC}_5Con$ between $\mathbb{A}_{\mathrm{if}}$ and $\mathbb{B}_{\mathrm{if}}$, since there exists no $\mathcal{IFOS}$ $\mathbb{E}_{\mathrm{if}}$ in $\mathbb{U}$ such that $\mathbb{A}_{\mathrm{if}} \subseteq \mathbb{E}_{\mathrm{if}}$ and $\mathbb{E}_{\mathrm{if}\ q}\ \mathbb{B}_{\mathrm{if}}$. But it is not $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}Con$ between $\mathbb{A}_{\mathrm{if}}$ and $\mathbb{B}_{\mathrm{if}}$, since there exists an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OS}$ $\mathbb{E}_{\mathrm{if}}$={<$\mathfrak{e}$, 0.7, 0.3>, <$\mathfrak{f}$, 0.8, 0.2>} such that $\mathbb{A}_{\mathrm{if}} \subseteq \mathbb{E}_{\mathrm{if}}$ and $\mathbb{E}_{\mathrm{if}\ q}\ \mathbb{B}_{\mathrm{if}}$.

Theorem 3.11. If an $\mathcal{IFTS}$ $(\mathbb{U},\tau_{if})$ is $\mathcal{IF}\hat{g}^*sCon$ between two $\mathcal{IFS}$s $\mathbb{G}_{if}$ and $\mathbb{H}_{if}$ and $\mathbb{G}_{if} \subseteq \mathcal{M}_{if}$, $\mathbb{H}_{if} \subseteq \mathcal{N}_{if}$, then $(\mathbb{U},\tau_{if})$ is $\mathcal{IF}\hat{g}^*sCon$ between $\mathcal{M}_{if}$ and $\mathcal{N}_{if}$.

Proof: Suppose that $(\mathbb{U},\tau_{if})$ is not $\mathcal{IF}\hat{g}^*sCon$ between $\mathcal{M}_{if}$ and $\mathcal{N}_{if}$, then by Def. 3.2., there exists an $\mathcal{IF}\hat{g}^*s\mathcal{OS}$ $\mathbb{I}_{if}$ in $(\mathbb{U},\tau_{if})$ such that $\mathcal{M}_{if} \subseteq \mathbb{I}_{if}$ and $\mathbb{I}_{if}\ {}_{q}{}^{c}\ \mathcal{N}_{if}$. This implies $\mathbb{I}_{if} \subseteq \mathcal{N}_{if}{}^{c}$.
$\mathcal{M}_{if} \subseteq \mathbb{I}_{if}$ implies $\mathbb{G}_{if} \subseteq \mathcal{M}_{if} \subseteq \mathbb{I}_{if}$. That is $\mathbb{G}_{if} \subseteq \mathbb{I}_{if}$.
Now let us prove that $\mathbb{I}_{if} \subseteq \mathbb{H}_{if}{}^{c}$, that is $\mathbb{I}_{if}\ {}_{q}{}^{c}\ \mathbb{H}_{if}$. Suppose $\mathbb{I}_{if}\ {}_{q}\ \mathbb{H}_{if}$, then by Def. 2.8., there exists an element $\mathbb{w}$□$\mathbb{U}$ such that $\mu_{\mathbb{I}_{if}}(\mathbb{w}) > \upsilon_{\mathbb{H}_{if}}(\mathbb{w})$ or $\upsilon_{\mathbb{I}_{if}} < \mu_{\mathbb{H}_{if}}(\mathbb{w})$. Therefore $\mu_{\mathbb{I}_{if}}(\mathbb{w}) > \upsilon_{\mathbb{H}_{if}}(\mathbb{w}) > \upsilon_{\mathcal{N}_{if}}(\mathbb{w})$ and $\upsilon_{\mathbb{I}_{if}} < \mu_{\mathbb{H}_{if}}(\mathbb{w}) < \mu_{\mathcal{N}_{if}}(\mathbb{w})$, since $\mathbb{H}_{if} \subseteq \mathcal{M}_{if}$. Hence $\mu_{\mathbb{I}_{if}}(\mathbb{w}) > \upsilon_{\mathcal{N}_{if}}(\mathbb{w})$ and $\upsilon_{\mathbb{I}_{if}} < \mu_{\mathcal{N}_{if}}(\mathbb{w})$. Thus $\mathbb{I}_{if}\ {}_{q}\ \mathcal{N}_{if}$, which is a contradiction. Therefore $\mathbb{I}_{if}\ {}_{q}{}^{c}\ \mathbb{H}_{if}$. Hence $(\mathbb{U},\tau_{if})$ is not $\mathcal{IF}\hat{g}^*sCon$ between two $\mathcal{IFS}$s $\mathbb{G}_{if}$ and $\mathbb{H}_{if}$, which is a contradiction to our hypothesis. Thus $(\mathbb{U},\tau_{if})$ must be $\mathcal{IF}\hat{g}^*sCon$ between $\mathcal{M}_{if}$ and $\mathcal{N}_{if}$.

Theorem 3.12. Let $(\mathbb{U},\tau_{if})$ be an $\mathcal{IFTS}$ and $\mathbb{G}_{if}$ and $\mathbb{H}_{if}$ be $\mathcal{IFS}$s in $(\mathbb{U},\tau_{if})$. If $\mathbb{G}_{if}\ {}_{q}\ \mathbb{H}_{if}$, then $(\mathbb{U},\tau_{if})$ is $\mathcal{IF}\hat{g}^*sCon$ between $\mathbb{G}_{if}$ and $\mathbb{H}_{if}$.

Proof: Suppose $(\mathbb{U},\tau_{if})$ is not $\mathcal{IF}\hat{g}^*sCon$ between $\mathbb{G}_{if}$ and $\mathbb{H}_{if}$. Then there exists an $\mathcal{IF}\hat{g}^*s\mathcal{OS}$ $\mathbb{I}_{if}$ in $(\mathbb{U},\tau_{if})$ such that $\mathbb{G}_{if} \subseteq \mathbb{I}_{if}$ and $\mathbb{I}_{if}\ {}_{q}{}^{c}\ \mathbb{H}_{if}$ then $\mathbb{I}_{if} \subseteq \mathbb{H}_{if}{}^{c}$. This implies $\mathbb{G}_{if} \subseteq \mathbb{H}_{if}{}^{c}$. That is $\mathbb{G}_{if}\ {}_{q}{}^{c}\ \mathbb{H}_{if}$. This contradicts our hypothesis. Therefore $(\mathbb{U},\tau_{if})$ must be $\mathcal{IF}\hat{g}^*sCon$ between $\mathbb{G}_{if}$ and $\mathbb{H}_{if}$.

Definition 3.13. An $\mathcal{IF}\hat{g}^*s\mathcal{OS}$ $\mathbb{G}_{if}$ is an $\mathcal{IF}$ regular $\hat{g}^*$ semi-open set ($\mathcal{IFR}\hat{g}^*s\mathcal{OS}$) if $\mathbb{G}_{if} = \hat{g}^*sint_{if}(\hat{g}^*scl_{if}(\mathbb{G}_{if}))$. The complement of an $\mathcal{IFR}\hat{g}^*s\mathcal{OS}$ is an $\mathcal{IFR}\hat{g}^*s\mathcal{CS}$.

Definition 3.14. An $\mathcal{IFTS}$ $(\mathbb{U},\tau_{if})$ is called an intuitionistic fuzzy $\hat{g}^*$ semi super connected ($\mathcal{IF}\hat{g}^*sSConS$) if there exists no proper $\mathcal{IFR}\hat{g}^*s\mathcal{OS}$ in $(\mathbb{U},\tau_{if})$.

Theorem 3.15. Let $(\mathbb{U},\tau_{if})$ be an $\mathcal{IFTS}$. Then the following statements are equivalent:
(a) $(\mathbb{U},\tau_{if})$ is an $\mathcal{IF}\hat{g}^*sSConS$.
(b) For every non-zero $\mathcal{IF}\hat{g}^*s\mathcal{OS}$ $\mathbb{G}_{if}$, $\hat{g}^*scl_{if}(\mathbb{G}_{if}) = \tilde{\mathbf{1}}$.
(c) For every $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ $\mathbb{G}_{if}$ with $\mathbb{G}_{if} \neq \tilde{\mathbf{1}}$, $\hat{g}^*sint(\mathbb{G}_{if}) = \tilde{\mathbf{0}}$.
(d) There exist no $\mathbb{G}_{if}$ and $\mathbb{H}_{if}$ in $(\mathbb{U},\tau_{if})$ such that $\mathbb{G}_{if} \neq \tilde{\mathbf{0}} \neq \mathbb{H}_{if}$, $\mathbb{G}_{if} \subseteq \mathbb{H}_{if}{}^{c}$.
(e) There exist no $\mathbb{G}_{if}$ and $\mathbb{H}_{if}$ in $(\mathbb{U},\tau_{if})$ such that $\mathbb{G}_{if} \neq \tilde{\mathbf{0}} \neq \mathbb{H}_{if}$, $\mathbb{H}_{if} = (\hat{g}^*scl_{if}(\mathbb{G}_{if}))^{c}$, $\mathbb{G}_{if} = (\hat{g}^*scl_{if}(\mathbb{H}_{if}))^{c}$.
(f) There exist no $\mathbb{G}_{if}$ and $\mathbb{H}_{if}$ in $(\mathbb{U},\tau_{if})$ such that $\mathbb{G}_{if} \neq \tilde{\mathbf{0}} \neq \mathbb{H}_{if}$, $\mathbb{H}_{if} = (\hat{g}^*sint_{if}(\mathbb{G}_{if}))^{c}$, $\mathbb{G}_{if} = (\hat{g}^*sint_{if}(\mathbb{H}_{if}))^{c}$.

Proof: (a) ⇒ (b) Assume that there exists an $\mathcal{IF}\hat{g}^*s\mathcal{OS}$ $\mathbb{G}_{if}$ in $(\mathbb{U},\tau_{if})$ such that $\mathbb{G}_{if} \neq \tilde{\mathbf{0}}$ and $\hat{g}^*scl_{if}(\mathbb{G}_{if}) \neq \tilde{\mathbf{1}}$. Now let $\mathbb{H}_{if} = \hat{g}^*sint_{if}(\hat{g}^*scl_{if}(\mathbb{G}_{if}))^{c}$. Then $\mathbb{H}_{if}$ is a proper $\mathcal{IFR}\hat{g}^*s\mathcal{OS}$ in $(\mathbb{U},\tau_{if})$, which contradicts the assumption. Therefore $\hat{g}^*scl_{if}(\mathbb{G}_{if}) = \tilde{\mathbf{1}}$.

(b) ⇒ (c) Let $\mathbb{G}_{if} \neq \tilde{\mathbf{1}}$ be an $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ in $(\mathbb{U},\tau_{if})$. If $\mathbb{H}_{if} = \mathbb{G}_{if}{}^{c}$, then $\mathbb{H}_{if}$ is an $\mathcal{IF}\hat{g}^*s\mathcal{OS}$ in $(\mathbb{U},\tau_{if})$ with $\mathbb{H}_{if} \neq \tilde{\mathbf{0}}$. Hence $\hat{g}^*scl_{if}(\mathbb{H}_{if}) = \tilde{\mathbf{1}}$. This implies $(\hat{g}^*scl_{if}(\mathbb{H}_{if}))^{c} = \tilde{\mathbf{0}}$. That is $\hat{g}^*sint_{if}(\mathbb{H}_{if}{}^{c}) = \tilde{\mathbf{0}}$. Hence $\hat{g}^*sint(\mathbb{G}_{if}) = \tilde{\mathbf{0}}$.

(c) ⇒ (d) Suppose $\mathbb{G}_{if}$ and $\mathbb{H}_{if}$ be two $\mathcal{IF}\hat{g}^*s\mathcal{OS}$s in $(\mathbb{U},\tau_{if})$ such that $\mathbb{G}_{if} \neq \tilde{\mathbf{0}} \neq \mathbb{H}_{if}$ and $\mathbb{G}_{if} \subseteq \mathbb{H}_{if}{}^{c}$. Then $\mathbb{H}_{if}{}^{c}$ is an $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ in $(\mathbb{U},\tau_{if})$ and $\mathbb{H}_{if} \neq \tilde{\mathbf{0}}$ implies $\mathbb{H}_{if}{}^{c} \neq \tilde{\mathbf{1}}$. By hypothesis $\hat{g}^*sint_{if}(\mathbb{H}_{if}{}^{c}) = \tilde{\mathbf{0}}$. But $\mathbb{G}_{if} \subseteq \mathbb{H}_{if}{}^{c}$. Therefore $\tilde{\mathbf{0}} \neq \mathbb{G}_{if} = \hat{g}^*sint(\mathbb{G}_{if}) \subseteq \hat{g}^*sint(\mathbb{H}_{if}{}^{c}) = \tilde{\mathbf{0}}$, which is a contradiction. Therefore (d) is true.

(d) ⇒ (a) Suppose $\tilde{\mathbf{0}} \neq \mathbb{G}_{if} \neq \tilde{\mathbf{1}}$ be an $\mathcal{IFR}\hat{g}^*s\mathcal{OS}$ in $(\mathbb{U},\tau_{if})$. If we take $\mathbb{H}_{if} = (\hat{g}^*scl_{if}(\mathbb{G}_{if}))^{c}$,
Then $\mathbb{H}_{if}$ is an $\mathcal{IFR}\hat{g}^*s\mathcal{OS}$, since $\hat{g}^*sint_{if}(\hat{g}^*scl_{if}(\mathbb{H}_{if})) = \hat{g}^*sint_{if}(\hat{g}^*scl_{if}(\hat{g}^*scl_{if}(\mathbb{G}_{if}))^{c})$
$= \hat{g}^*sint_{if}(\hat{g}^*sint_{if}(\hat{g}^*scl_{if}(\mathbb{G}_{if})))^{c} = \hat{g}^*sint_{if}(\mathbb{G}_{if}{}^{c}) = (\hat{g}^*scl_{if}(\mathbb{G}_{if}))^{c} = \mathbb{H}_{if}$. Also we get $\mathbb{H}_{if} \neq \tilde{\mathbf{0}}$, since otherwise if $\mathbb{H}_{if} = \tilde{\mathbf{0}}$ then this implies $(\hat{g}^*scl_{if}(\mathbb{G}_{if}))^{c} = \tilde{\mathbf{0}}$. That is $\hat{g}^*scl_{if}(\mathbb{G}_{if}) = \tilde{\mathbf{1}}$. Hence $\mathbb{G}_{if} = \hat{g}^*sint_{if}(\hat{g}^*scl_{if}(\mathbb{G}_{if})) = \hat{g}^*sint_{if}(\tilde{1})$ $= \tilde{\mathbf{1}}$, which is a contradiction. Therefore $\mathbb{H}_{if} \neq \tilde{\mathbf{0}}$ and $\mathbb{G}_{if} \subseteq \mathbb{H}_{if}{}^{c}$. But this is a contradiction to (d). Therefore $(\mathbb{U},\tau_{if})$ must be an $\mathcal{IF}\hat{g}^*s$ super connected space.

(a) ⇒ (e) Suppose $\mathbb{G}_{if}$ and $\mathbb{H}_{if}$ be two $\mathcal{IF}\hat{g}^*s\mathcal{OS}$s in $(\mathbb{U},\tau_{if})$ such that $\mathbb{G}_{if} \neq \tilde{\mathbf{0}} \neq \mathbb{H}_{if}$ and $\mathbb{H}_{if} = (\hat{g}^*scl_{if}(\mathbb{G}_{if}))^c$, $\mathbb{G}_{if} = (\hat{g}^*scl_{if}(\mathbb{H}_{if}))^c$. Now we have $\hat{g}^*sint_{if}(\hat{g}^*scl_{if}(\mathbb{G}_{if})) = \hat{g}^*sint(\mathbb{H}_{if}{}^c) = (\hat{g}^*scl_{if}(\mathbb{H}_{if}))^c = \mathbb{G}_{if}$, $\mathbb{G}_{if} \neq \tilde{\mathbf{0}}$ and $\mathbb{G}_{if} \neq \tilde{\mathbf{1}}$, since if $\mathbb{G}_{if} = \tilde{\mathbf{1}}$, then $(\hat{g}^*scl_{if}(\mathbb{H}_{if}))^c \Rightarrow \hat{g}^*scl_{if}(\mathbb{H}_{if}) = \tilde{\mathbf{0}} \Rightarrow \mathbb{H}_{if} = \tilde{\mathbf{0}}$. Therefore $\mathbb{G}_{if} \neq \tilde{\mathbf{1}}$. That is $\mathbb{G}_{if}$ is a proper $\mathcal{IFR}\hat{g}^*s\mathcal{OS}$ in $(\mathbb{U},\tau_{if})$, which is a contradiction to (a). Hence (e) is true.

(e) ⇒ (a) Let $\mathbb{G}_{if}$ be an $\mathcal{IF}\hat{g}^*s\mathcal{OS}$ in $(\mathbb{U},\tau_{if})$ such that $\tilde{\mathbf{0}} \neq \mathbb{G}_{if} \neq \tilde{\mathbf{1}}$. Now take $\mathbb{H}_{if} = (\hat{g}^*scl_{if}(\mathbb{G}_{if}))^c$. In this case we get $\mathbb{H}_{if} = \tilde{\mathbf{0}}$ and $\mathbb{H}_{if}$ is an $\mathcal{IF}\hat{g}^*s\mathcal{OS}$ in $(\mathbb{U},\tau_{if})$. Now $\mathbb{H}_{if} = (\hat{g}^*scl_{if}(\mathbb{G}_{if}))^c$ and $(\hat{g}^*scl_{if}(\mathbb{H}_{if}))^c = (\hat{g}^*scl_{if}(\hat{g}^*scl_{if}(\mathbb{G}_{if}))^c)^c = \hat{g}^*sint_{if}(\hat{g}^*scl_{if}(\mathbb{G}_{if})^c)^c$ $\hat{g}^*sint_{if}(\hat{g}^*scl_{if}(\mathbb{G}_{if})) = \mathbb{G}_{if}$. But this is a contradiction to (e). Therefore $(\mathbb{U},\tau_{if})$ must be an $\mathcal{IF}\hat{g}^*s$ super connected space.

(e) ⇒ (f) Suppose $\mathbb{G}_{if}$ and $\mathbb{H}_{if}$ be two $\mathcal{IF}\hat{g}^*s\mathcal{CS}$s in $(\mathbb{U},\tau_{if})$ such that $\mathbb{G}_{if} \neq \tilde{\mathbf{0}} \neq \mathbb{H}_{if}$, $\mathbb{H}_{if} = (\hat{g}^*sint_{if}(\mathbb{G}_{if}))^c$, $\mathbb{G}_{if} = (\hat{g}^*sint_{if}(\mathbb{H}_{if}))^c$. Taking $\mathbb{I}_{if} = \mathbb{G}_{if}{}^c$ and $\mathbb{J}_{if} = \mathbb{H}_{if}{}^c$, $\mathbb{I}_{if}$ and $\mathbb{J}_{if}$ become $\mathcal{IF}\hat{g}^*s\mathcal{OS}$s in $(\mathbb{U},\tau_{if})$ with $\mathbb{I}_{if} \neq \tilde{\mathbf{0}} \neq \mathbb{J}_{if}$ and $\mathbb{J}_{if} = (\hat{g}^*scl_{if}(\mathbb{I}_{if}))^c$, $\mathbb{I}_{if} = (\hat{g}^*scl_{if}(\mathbb{J}_{if}))^c$, which is a contradiction to (e). Hence (f) is true.

(f) ⇒ (e) can be proved easily by the similar way as in (e) ⇒ (f).

Definition 3.16. An $\mathcal{IFTS}$ $(\mathbb{U},\tau_{if})$ is called an intuitionistic fuzzy $\hat{g}^*$ semi extremally disconnected ($\mathcal{IF}\hat{g}^*EDconS$) if the $\hat{g}^*s$ closure of every $\mathcal{IF}\hat{g}^*s\mathcal{OS}$ in $(\mathbb{U},\tau_{if})$ is $\mathcal{IF}\hat{g}^*s\mathcal{OS}$.

Theorem 3.17. Let $(\mathbb{U},\tau_{if})$ be an $\mathcal{IFTS}$. Then the following statements are equivalent:
(a) $(\mathbb{U},\tau_{if})$ is an $\mathcal{IF}\hat{g}^*sEDconS$.
(b) For each $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ $\mathbb{G}_{if}$, $\hat{g}^*sint_{if}(\mathbb{G}_{if})$ is an $\mathcal{IF}\hat{g}^*s\mathcal{CS}$
(c) For each $\mathcal{IF}\hat{g}^*s\mathcal{OS}$ $\mathbb{G}_{if}$, $\hat{g}^*scl_{if}(\mathbb{G}_{if}) = (\hat{g}^*scl_{if}(\hat{g}^*scl_{if}(\mathbb{G}_{if}))^c)^c$
(d) For each pair of $\mathcal{IF}\hat{g}^*s\mathcal{OS}$s $\mathbb{G}_{if}$ and $\mathbb{H}_{if}$ with $\hat{g}^*scl_{if}(\mathbb{G}_{if}) = \mathbb{H}_{if}{}^c$ implies that $\hat{g}^*scl_{if}(\mathbb{G}_{if}) = (\hat{g}^*scl_{if}(\mathbb{H}_{if}))^c$.

Proof: (a) ⇒ (b) Let $\mathbb{G}_{if}$ be any $\mathcal{IF}\hat{g}^*s\mathcal{CS}$. Then $\mathbb{G}_{if}{}^c$ is an $\mathcal{IF}\hat{g}^*s\mathcal{OS}$. So (a) implies that $\hat{g}^*scl_{if}(\mathbb{G}_{if}{}^c) = (\hat{g}^*sint_{if}(\mathbb{G}_{if}))^c$ is an $\mathcal{IF}\hat{g}^*s\mathcal{OS}$. Thus $\hat{g}^*sint_{if}(\mathbb{G}_{if})$ is an $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ in $(\mathbb{U},\tau_{if})$.

(b)⇒(c) Let $\mathbb{G}_{if}$ be any $\mathcal{IF}\hat{g}^*s\mathcal{OS}$. Then we have $\hat{g}^*scl_{if}(\hat{g}^*scl_{if}(\mathbb{G}_{if}))^c = \hat{g}^*scl_{if}(\hat{g}^*sint_{if}(\mathbb{G}_{if})^c)$. Therefore $(\hat{g}^*scl_{if}(\hat{g}^*scl_{if}(\mathbb{G}_{if}))^c)^c = (\hat{g}^*scl_{if}(\hat{g}^*sint_{if}(\mathbb{G}_{if}{}^c)))^c$. Since $\mathbb{G}_{if}$ is an $\mathcal{IF}\hat{g}^*s\mathcal{OS}$, $\mathbb{G}_{if}{}^c$ is $\mathcal{IF}\hat{g}^*s\mathcal{CS}$. So by (b), $\hat{g}^*sint_{if}(\mathbb{G}_{if}{}^c)$ is an $\mathcal{IF}\hat{g}^*s\mathcal{CS}$. That is $\hat{g}^*scl_{if}(\hat{g}^*sint_{if}(\mathbb{G}_{if}{}^c)) = \hat{g}^*sint_{if}(\mathbb{G}_{if}{}^c)$. Hence $(\hat{g}^*scl_{if}(\hat{g}^*scl_{if}(\mathbb{G}_{if}))^c)^c = (\hat{g}^*sint_{if}(\mathbb{G}_{if}{}^c))^c = \hat{g}^*scl_{if}(\mathbb{G}_{if})$.

(c) ⇒ (d) Let $\mathbb{G}_{if}$ and $\mathbb{H}_{if}$ be any two $\mathcal{IF}\hat{g}^*s\mathcal{OS}$s in $(\mathbb{U},\tau_{if})$ such that $\hat{g}^*scl_{if}(\mathbb{G}_{if}) = \mathbb{H}_{if}{}^c$. (c) implies $\hat{g}^*scl_{if}(\mathbb{G}_{if}) = (\hat{g}^*scl_{if}(\hat{g}^*scl_{if}(\mathbb{G}_{if}))^c)^c = (\hat{g}^*scl_{if}(\mathbb{H}_{if}{}^c)^c)^c = (\hat{g}^*scl_{if}(\mathbb{H}_{if}))^c$.

(d) ⇒ (a) Let $\mathbb{G}_{if}$ be an $\mathcal{IF}\hat{g}^*s\mathcal{OS}$ in $(\mathbb{U},\tau_{if})$. Put $\mathbb{H}_{if} = (\hat{g}^*scl_{if}(\mathbb{G}_{if}))^c$, then $\mathbb{H}_{if}$ is an $\mathcal{IF}\hat{g}^*s\mathcal{OS}$ and $\hat{g}^*scl_{if}(\mathbb{G}_{if}) = \mathbb{H}_{if}{}^c$. Hence by (d), $\hat{g}^*scl_{if}(\mathbb{G}_{if}) = (\hat{g}^*scl_{if}(\mathbb{H}_{if}))^c$. Since $\hat{g}^*scl_{if}(\mathbb{H}_{if})$ is an $\mathcal{IF}\hat{g}^*s\mathcal{CS}$, it follows that $\hat{g}^*scl_{if}(\mathbb{G}_{if})$ is $\mathcal{IF}\hat{g}^*s\mathcal{OS}$. This implies that $(\mathbb{U},\tau_{if})$ is an $\mathcal{IF}\hat{g}^*sEDconS$.

References

[1] Atanassov K, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, 20, 1986, 87-96.
[2] Chang C. L, *Fuzzy topological spaces*, J.Math.Anal.Appl. 24, 1968, 182-190.
[3] Coker D, *An introduction to intuitionistic fuzzy topological space*, Fuzzy Sets and Systems, 88, 1997, 81-89.
[4] Gurcay H, Haydar A and Coker D, *On fuzzy continuity in intuitionistic fuzzy topological spaces*, The J.Fuzzy Math.5 (2) , 1997, 365-378.
[5] Pious Missier S, Peter Arokiaraj A, A. Nagarajan & S.Jackson, Intuitionistic Fuzzy $\hat{g}^*$ Semi Closed and Open Sets in Intuitionistic Fuzzy Topological Spaces (Communicated).
[6] Pious Missier S, Peter Arokiaraj A and Jackson S, Intuitionistic Fuzzy $\hat{g}^*$ Semi Continuous Mappings and Irresolute Function in Intuitionistic Fuzzy Topological Spaces, Turkish Journal of Computer and Mathematics Education, 2020, Vol.11, No.3 ISSN 1039-4653, pp. 2601-2609.
[7] Turnali N and Coker D, *Fuzzy connectedness in intuitionistic fuzzy topological spaces*, FuzzySets and Systems 116, 2000, 369–375.
[8] Zadeh L.A, *Fuzzy sets*, Information and control, 8, 1965, 338-353.