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Integral Solutions Of Homogeneous Quadratic Equation $5x^2 + 5y^2 - 6XY = 13z^2$

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Abstract:
The ternary homogeneous quadratic Diophantine equation is analyzed for finding its non-zero distinct integral solutions. Seven different patterns of integer solutions are presented. A few interesting relations between the solutions are presented. Introducing the linear transformation x=u+v, y=u-v and employing the method of factorization,different patterns of non-zero distinct integer solutions to the above equations are obtained. Also by using specific transformations, various pattern of solutions are exhibited.

Keywords: Ternary quadratic, homogeneous quadratic, integer solutions.
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I.INTRODUCTION

The Ternary Quadratic Diophantine Equation offers an unlimited field for research because of their variety [1-2]. For an extensive review of various problems, one may refer [3-10]. This communication concerns with yet another interesting ternary Quadratic equation $5x^2 + 5y^2 - 6XY = 13z^2$ for determining its infinitely many non-zero integral solutions. Also a few interesting relations among the solutions have been presented..

Notations used :
t m,n= Polygonal number of rank n with size m.
obl_n = oblong number of rank n

II.METHOD OF ANALYSIS

The Ternary Quadratic Diophantine with three unknowns to be solved for getting non – zerointegral solution is $5x^2 + 5y^2 - 6XY = 13z^2$ ------------(1)
We present below different patterns of non-zero distinct integer solutions to (1)

Pattern : 1
Introducing the linear transformations
$x = u + v$; $y = u - v$ ………………… (2) , it leads to

$u^2 + 4v^2 = 13z^2$ --------(3)

Assume $z = z(a,b) = a^2 + 4b^2$ ------(4)

Also write 13 as

$13 = (3 + i\sqrt{4})(3 - i\sqrt{4})$ -------------(5)

$u^2 + 4v^2 = (u + i\sqrt{4}\, v)(u - i\sqrt{4}\, v)$ -------(6)

Substitute (4) and (4) in (3) and employing the method of factorization, define

$u + i\sqrt{4}v = (3 + i\sqrt{4}\, v)(a + i\sqrt{b}\,)^2$

Equating the real and imaginary parts, we get

$u = 3a^2 + \grave{}12b^2 - 8ab$

$v = a^2 + \grave{}4b^2 + 6ab$

Using this transformation in (1), it leads to the following set of integral solutions:

$x = x(a,b) = u + v = 4a^2 - 16b^2 - 2ab$

$y = y(a,b) = u - v = 2a^2 - 8b^2 - 14ab$

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$z = z(a,b) = a^2 + 4b^2$

Properties:

$x(1,n) - y(n,1) is\ a\ \ nasty\ \ number$

$x(10,n) - y(30,n) \equiv 0 \pmod{100}$

$x(1,n) - z(1,n) = -2ob\ln$

Pattern : 2

Equation (1) can also be written as

$u^2 - 9z^2 = 4(z^2 - v^2)$

$(u + 3z)(u - 3z) = 4(z + v)(z - v)$ -------------(7),

which may be written in the form of ratio as

$(u + 3z)/(z + v) = 4(z - v)/(u - 3z)$, where $\frac{A}{B} \neq 0$

On employing the method of cross multiplication, we get

$u = -3A^2 + 12B^2 - 8AB$

$v = A^2 - 4B^2 - 6AB$

$z = z(a,b) = -4B^2 - A^2$

Using this transformation in (1), it leads to the following set of integral solutions:

$x = x(a,b) = u + v = -2A^2 + 8B^2 - 14AB$

$y = y(a,b) = u - v = -4A^2 + 16B^2 - 2AB$

$z = z(a,b) = -a^2 - 4b^2$

Properties:

$x(3,n) + z(n,6) - 9n^2\ is\ a\ \ nasty\ \ number$

$y(n,5) - z(5,n) \equiv 0 \pmod 5$

$x(3,n) - y(n,1) \equiv 0 \pmod 2$

It is observed that by rewriting (7) suitably, one may arrive at the following three patterns of solutions to (1).

Pattern : 3

$x = x(A,B) = u + v = 4A^2 - 8B^2 - 6AB$

$y = y(A,B) = u - v = 2A^2 - 16B^2 + 6AB$

$z = z(A,B) = -A^2 + 4B^2$

Properties:

$x(4,n) + y(n,1) \equiv 0 \pmod 2$

$x(n,1) - z(2,n) \equiv 0 \pmod 4$

$z(2,n) - y(1,n) \equiv 0 \pmod 2$

Pattern : 4

$x = x(A,B) = u + v = 4A^2 - 16B^2 + 2AB$

$y = y(A,B) = u - v = 12A^2 - 8B^2 - 14AB$

$z = z(A,B) = -A^2 - 4B^2$

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Properties:

$x(20,n)+y(20,n)$ *is a nasty number*

$y(40,n)+z(60,n) \equiv 0 \pmod{20}$

$x(n,1)+y(n,1) - 16n$ *is a nasty number*

Pattern : 5

$$x = x(A,B) = u + v = -2A^2 + 8B^2 + 14AB$$

$$y = y(A,B) = u - v = -4A^2 + 16B^2 + 2AB$$

$$z = z(A,B) = A^2 + 4B^2$$

Properties:

$y(n,2)+z(2,n) - 4n$ *is a nasty number*

$z(2,n)+y(n,1) \equiv 0 \pmod{2}$

$x(1,n)+y(1,n)$ *is a nasty number*

Pattern : 6

Equation (1) can also be written as

$4v^2 = 13z^2 - u^2$ --- -----------(8)

Introducing the transformation

$v = 13a^2 - b^2$ ----------------(9) and

Writing 4 as $4 = (2\sqrt{13} - 4\sqrt{13})(2\sqrt{13} + 4\sqrt{13})$ -------------(10)

Using (9) and (10) in (8) and employing the method of factorization, define

$\sqrt{13}z + u = -26\sqrt{13}a^2 - 2\sqrt{13}b^2 + 26ab$ ----------------(11)

Equating rational and irrational parts in (8), we get

$$u = 26ab$$

$$v = 13a^2 - b^2$$

$$z = z(A,B) = -26a^2 - 2b^2$$

Using this transformation in (1), it leads to the following set of integral solutions:

$$x = x(A,B) = u + v = 13A^2 - B^2 + 26AB$$

$$y = y(A,B) = u - v = -13A^2 + B^2 + 26AB$$

$$z = z(A,B) = 26A^2 - 2B^2$$

Properties:

$x(1,n) - y(1,n) = obl3$

$x(1,n) - y(1,n) + 5n^2 \equiv 0 \pmod{2}$

$x(1,n)+y(1,n)$ *is a nasty number*

Pattern : 7

Introducing the transformation

$z = a^2 + 4b^2$ --------(12), equation (3) takes the form

$(u + i\sqrt{4}v)(u - i\sqrt{4}v) = 13(a^2 + 4b^2)^2$ --------(13)

Equating rational and irrational parts in (13), we get

$$u = 3a^2 - 12b^2 - 8ab$$

$$v = -a^2 - 4b^2 + 6ab$$

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$z = a^2 + 4b^2$

Using this transformation in (1), it leads to the following set of integral solutions:

$x = x(a,b) = u + v = 2a^2 - 16b^2 - 2ab$

$y = y(a,b) = -a^2 - 4b^2 + 6ab$

$z = z(a,b) = a^2 + 4b^2$

Properties:

$x(1,n) - z(1,n) = 0 \pmod 8$

$x(n,1) - z(2,n) \equiv 0 \pmod 4$

$x(1,n) + y(1,n)$ *is a nasty number*

CONCLUSION:

The ternary homogeneous quadratic Diophantine equation is analyzed for finding its non-zero distinct integral solutions. Seven different patterns of integer solutions were presented along with few properties.

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