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Global Stability Analysis of Rumor Propagation in Social Networks

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ABSTRACT

Rumor propagation is a typical form of social interaction that has a negative effect on society and human life. The study of rumor dissemination and regulation on social networks is significant and vital. The propagation of rumors is caused by a variety of circumstances. In this study, we take into account how people react to rumors differently depending on their personalities and levels of comprehension. In order to categorize the entire human population, we use the terms susceptible (S), exposed (E), infected (I), and recovered (R).

The dynamics of rumor transmission in social networks are represented by differential equations. The spreading threshold of the SEIR model in social networks is then obtained using the Jacobian matrix and the Next Generation matrix. Then the equilibrium's existence and stability are analyzed. It is established that rumor-free equilibrium E_0 is locally asymptotically stable if the basic reproduction number is low, which means rumor has stopped spreading in a population, and unstable if the basic reproduction number is high, which means new rumor is spreading in the population. Finally, in order to validate the analytic findings, numerical simulations of the dynamic model are run on the system using MATLAB.

Keywords: epidemic model, equilibrium point, reproduction number, stability

1. INTRODUCTION

News manipulation has been a significant endeavor for nearly everyone since ancient times. In the olden days, it was used to be done orally, but nowadays, the use of social networks, both online and offline, is common. Manipulated news or information, or rather, rumor, has discovered a strong medium called the Internet for its rapid and simple transmission. Because of their validity, confirmed or unconfirmed information is difficult to accept, but it has a significant influence on social networks and daily life [1–2].

In recent years, the popularity of online social media has increased dramatically. People can communicate more easily through social media. Meanwhile, the strategy allows untrustworthy sources to disseminate enormous volumes of unconfirmed material to individuals [3–7]. Rumours can thus spread more swiftly and broadly via online social media than through conventional offline social networks. The widespread transmission of disinformation may cause chaos in people's lives, especially when they are dealing with religious or political issues. This suggests that it is critical for social media to recognize disinformation in real time in order to restrict the spread of rumors [8]. Social communication has become incredibly quick and simple due to the development of social networking applications like blogs, instant messengers, social networking websites (like Facebook, Renren, and Wechat in China, and Vkontakte in Russia), professional networking websites (like LinkedIn), microblogging platforms (like Twitter and Weibo in China), and virtual worlds (like Second Life) [9].

Generally speaking, rumors are described as unsubstantiated elaborations or annotations of public items, events, or concerns that are created and spread through a variety of media, which can lead to unwarranted public worry and economic loss for the affected nations. What's worse is that rumors have spread more swiftly and extensively as a result of the advent of network connections [10]. As a result, understanding the theory behind rumor spreading is crucial since it may help manage and restrain the spread of rumor.

Researchers' interest in the transmission of rumor in intricate social networks has lately increased [11]. They establish the features and examine the crucial threshold of rumor propagation in the complex networks by describing the dynamics of the models using matching mean-field equations.

They establish the features and examine the crucial threshold of rumor propagation in the complex networks by describing the dynamics of the models using matching mean-field equations. The first conventional DK rumor propagation model was introduced in 1965 by Daley and Kendall [12]. Researchers studied the rumor using a mathematical model and created another formal model. Early classical models of rumor dynamics made the assumption that everyone had a uniform chance of connection [13–16]. These straightforward models obviously fall short of

accurately capturing the genuine characteristics of the spread of rumor, and in recent years, efforts have been made to improve their realism.
The paper is organized in the following way: Introduction is given in Section 1; model formulation and basic assumptions are given in Section 2; Section 3 establishes the stability of the system developed; numerical simulations are given in Section 4; and finally, a conclusion in Section 5.

2. MODEL FORMULATION

Thispaper establishes a model for the analysis of rumor spreading on online social networks with varying total populations.Online social networkingis a common platform for rumorspreading. So, in order to study the characteristics of rumorspreading in online social networks, we classify the network users into four categories,namely,susceptible group denoted by (S), Exposed group denoted by (E), Infected group denoted by (I),and lastly protestors group denoted by (R).
The total number of online users at time t can be denoted by N(t).Similarly the whole social network population density at time ‘ t’ can be denoted as $S(t), E(t), I(t), and\ R(t)$. Thus, we can say that
$N(t) = S(t) + E(t) + I(t) + R(t)$.Each of these four categories can be defined as follows:
-S $(Susceptible)$ represents those social network users, who are not yet infected by the news but are likely to be infected at any time by individuals at E(t) or I(t).Susceptibles are removed through death or exist from social network and is represented by μ.
-E $(Exposed)$ represents the number of those users who have received the news but may not have noticed and are likely to browse it at any time and hence are taking time to propagate the news.
-I $(Infected)$ represents those users who have received the news, read it and are sharing the rumor intentionally.
-R $(Recovered)$ represents those people who have seen or heard the rumor but want to verify the authenticity of the matter.If they find the rumor authentic, then only they propagate the message, otherwise they take measures to stop the spread of unauthentic rumor.

The model of transmission of rumor among these categories of people are shown in figure 1.

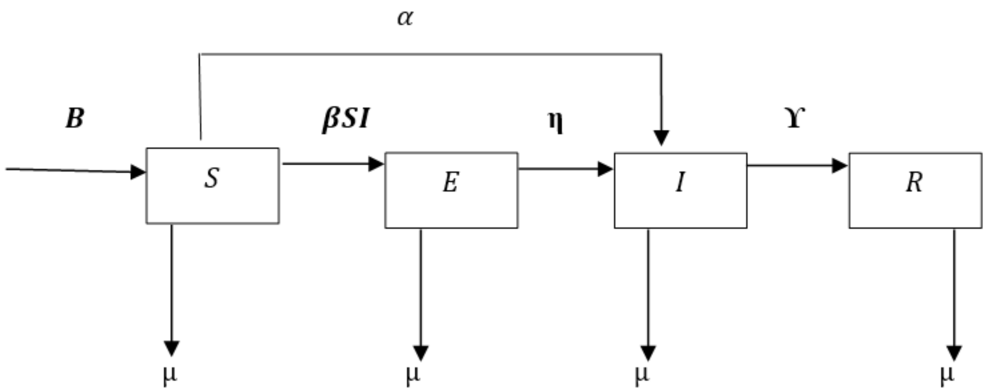

Figure 1: Model Diagram

Table 1:Model parameters and their explanation:

Parameters	Description
α	Transmission probability of rumor from susceptible human (S) to Infected human(I).
β	Transmission probability of rumor from susceptible human (S) to exposed human(E).
μ	Deactivation rate of rumor.
η	Transmission probability of rumor from exposed human (E) to infected human(I).
γ	Transmission probability of rumor from infected human (I) to recovered human(R).
B	Birth / Recruitment rate of rumorinto the susceptible class.

Thus,the differential equations for the model are as follows:

$$\frac{dS}{dt} = B - \beta SI - \mu S - \alpha S \qquad (1.1)$$

$$\frac{dE}{dt} = \beta SI - \mu E - \eta E \qquad (1.2)$$

$$\frac{dI}{dt} = \eta E + \alpha S - \mu I - \gamma I \qquad (1.3)$$

$$\frac{dR}{dt} = \gamma I - \mu R \qquad (1.4)$$

Basic Properties:
All of the variables and their corresponding parameters are non-negative all the time since the model (1.1) to (1.4) tracks human populations. It is crucial to demonstrate that the model's model variables are non-negative under all possible beginning circumstances.

Lemma 1:
Positive invariance characterises the area $D = \{(S, E, I, R) \in R_+^4 : S + E + I + R \leq \frac{B}{\mu}\}$which draws all solutions in R_+^4 .

Proof:
The rate of change of the entire human population may be calculated by adding all the equations from (1.1) to (1.4).

$$\frac{dN}{dt} = B - \mu N$$

Since $\frac{dN}{dt} = B - \mu N$,when $N(t) > \frac{B}{\mu} then \frac{dN}{dt} < 0$, indicating that$\frac{dN}{dt}$ is limited by $B - \mu N$.
The usual comparison theorem by [11] can be utilised to demonstrate that

$$N(t) \leq N(0)e^{-\mu t} + \frac{B}{\mu}(1 - e^{-\mu t})$$

.If$N(0) \leq \frac{B}{\mu}$, then$N(t) \leq \frac{B}{\mu}$. Thus, R is positively invariant (i.e., all solutions in D remain in D indefinitely). Furthermore, if $N(t) > \frac{B}{\mu}$, either the solution enters R in finite time or N(t) approaches $\frac{B}{\mu}$while the spreader variables E and I approach zero. As a result, D attracts solutions fromR_+^4. As a result, the model is epidemiologically and mathematically well formulated as all the variables are non-negative when t ≥ 0. Thus, it is sufficient to investigate the dynamics of the system (1.1) to (1.4) in D.

Stability of the model
In this section,we find the equilibrium states and basic reproduction number of the model.We also prove that our model is locally and globally stable for both rumor –free – equilibrium and endemic equilibrium points.Finding equilibrium states by setting the right hand side of all the model equations of system equal to zero,we obtain two equilibrium states:
(i) Rumour Free Equilibrium (RFE) state: E_0=($\frac{B}{\mu}$,0,0,0)
(ii) Endemic equilibrium state: E_1= (S*,E*,I*,R*)
Different sorts of behaviour are anticipated in the long run from the system under model. The equilibrium points and the conditions for their existence are that they provide us mathematical conditions based on which the long-term behaviour of the system can be predicted and classified into a finite number of possibilities represented by the equilibrium points.
Endemic Equilibrium points of the system (1.1 to 1.4):
From the first, second,third and fourth equation of the system and by equating to zero and solving it, we get

$$S^* = \frac{(\mu + \alpha)(\mu + \eta)(\mu + \gamma)}{(\mu + \alpha)\beta\eta + \alpha(B - (\mu + \eta))\beta}$$

$$E^* = \frac{(\mu + \alpha)(\mu + \eta)(\mu + \gamma)(\mu + \alpha)}{[B - (\mu + \eta)][(\mu + \alpha)\beta\eta + \alpha(B - (\mu + \eta))\beta]}$$

$$I^* = \frac{(\mu + \alpha)(\mu + \eta)}{(B - (\mu + \eta))\beta}$$

$$R^* = \frac{\gamma[(\mu+\alpha)(\mu+\eta)]}{\beta\mu[B-(\mu+\eta)]}$$

Basic Reproduction Number:
For any epidemic model, the basic reproduction number is the average number of secondary infectious cases produced by a single infection in total susceptible population. The basic reproduction number is calculated by R_0=ρ(FV^{-1}), where ρ is spectral radius of the matrix FV^{-1} and F&V are the matrices of new spreaders into the social network and rate of transfer into (out) of the social network respectively.

In this part, we analyse the model's existence and uniqueness, as well as its analysis of Rumour Free Equilibrium (RFE). The RFE of the model Equations (1.1) to (1.4) is provided by E_0=(S*,E*,I*,R*) = ($\frac{B}{\mu}$,0,0,0).

The supplied RFE's local stability will be explored using the next generation matrix approach. The next generation matrix for the system of questions (1.1) to (1.4) is calculated by counting the number of ways that:

1) New spreaders emerge
2) There are several methods for people to migrate, but only one technique to establish a spreader. So, let

F represent the rate of fresh rumorspreaders,and

V represent the rate of transfer in and out of the social media.

$$F=\begin{bmatrix} 0 & \beta \\ 0 & 0 \end{bmatrix}$$

$$V=\begin{bmatrix} -(\mu+\eta) & 0 \\ \eta & -(\gamma+\mu) \end{bmatrix}$$

$$\text{So, } V^{-1}=\begin{bmatrix} \frac{-1}{(\mu+\eta)} & 0 \\ \frac{-\eta}{(\mu+\eta)(\gamma+\mu)} & \frac{-1}{(\gamma+\mu)} \end{bmatrix}$$

Hence, the next generation matrix is two dimensional and is given by FV^{-1}.

$$FV^{-1}=\begin{bmatrix} \frac{-\eta\beta}{(\mu+\eta)(\gamma+\mu)} & \frac{-\beta}{(\gamma+\mu)} \\ 0 & 0 \end{bmatrix} \qquad (1.5)$$

The dominant eigenvalue of FV^{-1}=R_0, therefore,we evaluate the characteristic equation of $FV^{-1}$by using the$|FV^{-1}-\lambda I|=0$, which gives

$$R_0=\frac{\eta\beta}{(\mu+\eta)(\gamma+\mu)} \qquad (1.6)$$

Theorem 1:

The rumor-free equilibrium E_0 of the model equations (1.1) and (1.4) is locally asymptotically stable if $R_0<1$ and unstable if $R_0>1$. As a result, theorem 1 indicates that every given rumour in social networks may be removed when$R_0<1$.

Proof:

The Jacobian matrix of equations (1.1) to (1.4) is as follows:

$$J=\begin{bmatrix} -(\mu+\alpha) & 0 & 0 & 0 \\ 0 & -(\mu+\eta) & 0 & 0 \\ \alpha & \eta & -(\mu+\gamma) & 0 \\ 0 & 0 & \gamma & -\mu \end{bmatrix} \qquad (1.7)$$

Now, we try to find the eigenvalues from the Jacobian matrix (1.7) by finding the characteristic equation using the formula $|J-\lambda I|$=0

$$\begin{bmatrix} -(\mu+\alpha+\lambda) & 0 & 0 & 0 \\ 0 & -(\mu+\eta+\lambda) & 0 & 0 \\ \alpha & \eta & -(\mu+\gamma+\lambda) & 0 \\ 0 & 0 & \gamma & -(\mu+\lambda) \end{bmatrix}=0 \qquad (1.8)$$

Solving (1.8) ,we obtain

$$\lambda_1=-(\mu+\alpha)$$
$$\lambda_2=-(\mu+\eta)$$
$$\lambda_3=-(\mu+\gamma)$$
$$\lambda_4=-\mu$$

Since all the eigenvalues i.e. λ_1,λ_2, $\lambda_3$and λ_4 have negative real parts when$R_0<1$,we conclude that rumor-free equilibrium is locally asymptotically stable.

3. GLOBAL STABILITY FOR ENDEMIC EQUILIBRIUM:
In this section, we prove the global stability for endemic equilibrium. We adopt the geometrical approach for the mapping $g: L \subset R^n \rightarrow R^n$, where L is an open set, if its differential equations $y' = g(y)$ be such that its every solution $y(t)$ can be uniquely determined by its initial condition $y(t) = y_0$, then an equilibrium points $\bar{y} \in L$ and satisfies the following hypotheses
H1: L is simply connected
H2: There exists a compact absorbing sub set K of L

$\bar{x}$ the only equilibrium point in L is globally stable, if it satisfies the additional Bendixson condition given by $\bar{q}_2 = lim_{t\to\infty}^{sup} \max_{x_0 \in K} \frac{1}{t}\int_0^t \varphi\left(B\left(y(s,y_0)\right)\right)ds < 0$, where $B = P_f P^{-1} + P_f J^{[2]} P^{-1}$ and P is a matrix valued function satisfying $\varphi(P_f P^{-1} + P_f J^{[2]} P^{-1}) < 0$. Further $J^{[2]}$ is the second compound additive matrix of order three and φ denote the Lozinskii measure defined as $\varphi(B) = \lim_{h\to 0} \frac{|I+hB|-I}{h}$.
The existence of a compact absorbing set which is absorbing in the interior of region follows from the uniform persistence of the system as $\underset{t\to\infty}{lim} inf S(t) > \epsilon$, $\underset{t\to\infty}{lim} inf I(t) > \epsilon$, $\underset{t\to\infty}{lim} inf Q(t) > \epsilon$, $\underset{t\to\infty}{lim} inf R(t) > \epsilon$ for some$\epsilon > 0$. Based on this procedure system (1.1 to 1.4) is used to prove for Bendixsoncondition$\bar{q}_2 < 0$.

Theorem 2: The unique endemic equilibrium point E_1 is globally asymptotically stable if $R_0 > 1$.

Proof : For the general solution $\left(S(t), E(t), I(t)\right)$ of system , the Jacobian matrix is

$$J = \begin{bmatrix} -(\beta I + \mu + \alpha) & -\beta S & 0 \\ \beta I & -(\mu + \eta) & \beta S \\ \alpha & \eta & -(\mu + \gamma) \end{bmatrix}$$

The matrix$J^{[2]}$, the second additive compound matrix of the Jacobian for n=3, is defined as

$$J^{[2]} = \begin{bmatrix} j_{11} + j_{22} & j_{23} & -j_{13} \\ j_{32} & j_{11} + j_{33} & j_{12} \\ -j_{31} & j_{21} & j_{22} + j_{33} \end{bmatrix}.$$

So, its second additive compound matrix $J^{[2]}$ is

$$J^{[2]} = \begin{bmatrix} x & \beta S & \beta S \\ \beta S & y & \beta I \\ \beta S & 0 & -(\mu + \eta) - (\mu + \gamma) \end{bmatrix},$$

Where $x = -(\beta I + \mu + \alpha) - (\mu + \eta)$ and$y = -(\beta I + \mu + \alpha) - (\mu + \gamma)$.

Let the function $P = P(S, E, I)$ be defined as$P = P(S, E, I) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{E}{I} & 0 \\ 0 & 0 & \frac{E}{I} \end{bmatrix} = diag\left\{1, \frac{E}{I}, \frac{E}{I}\right\}$

$$\text{Then, } P_f P^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{E'}{E} - \frac{I'}{I} & 0 \\ 0 & 0 & \frac{E'}{E} - \frac{I'}{I} \end{bmatrix} \qquad (1.9)$$

WhereP_f is the matrix obtained by replacing each elements of P by its derivative in the direction of f.

$$P_f J^{[2]} P^{-1} = \begin{bmatrix} x & \beta S & \beta S \\ \beta S \frac{E}{I} & y + \frac{E'}{E} - \frac{I'}{I} & \beta I \\ BS & 0 & -(\mu + \eta) - (\mu + \gamma) + \frac{E'}{E} - \frac{I'}{I} \end{bmatrix},$$

Where$x = -(\beta I + \mu + \alpha) - (\mu + \eta)$ and$y = -(\beta I + \mu + \alpha) - (\mu + \gamma)$.

$$B = P_f P^{-1} + P_f J^{[2]} P^{-1} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix},$$

Where$B_{11} = [-(\beta I + \mu + \alpha) - (\mu + \eta)]$,$B_{12} = [\beta S \beta S]$, $B_{21} = \begin{bmatrix} \beta S \\ \beta S \end{bmatrix}$ and

$$B_{22} = \begin{bmatrix} -(\beta I + \mu + \alpha) - (\mu + \gamma) + \frac{E'}{E} - \frac{I'}{I} & \beta I \\ 0 & -(\mu + \eta) - (\mu + \gamma) + \frac{E'}{E} - \frac{I'}{I} \end{bmatrix}.$$

Now, for a vector (u, v, w) in $\boldsymbol{R}^3$, we select a norm as $|(u, v, w)| = max\{|u|,\ |v + w|\}$ and denote $\varphi(B)$ the Lozinskii measure for this norm.
From (1.9), it follows that $\varphi(B) \le \sup\{k_1, k_2\}$ (1.10)
Wherek_1 and k_2 are defined as follows:
$k_1 = B_{11} + |B_{12}|$ and$k_2 = \varphi_1(B_{22}) + |B_{21}|$, where $|B_{12}|$ and $|B_{21}|$ are matrix norms with respect to the vector norm L^1 and φ_1 denotes the Lozinskii measure with respect to the vector norm L^1. So, we have

$$k_1 = B_{11} + |B_{12}|$$
$$= -(\beta I + \mu + \alpha) - (\mu + \eta) + Sup\{\beta S, \beta S\}$$
$$k_1 = -(\beta I + \mu + \alpha) - (\mu + \eta) + \beta S \qquad (1.11)$$

Similarly, $k_2 = \varphi_1(B_{22}) + |B_{21}|$

$$= -(\mu + \gamma) - (\mu + \eta) - (\beta I + \mu + \alpha) + \frac{E'}{E} - \frac{I'}{I} + \beta S \frac{E}{I} \qquad (1.12)$$

From second and third equations of system (1.1 to 1.4), we can rewrite as

$$\frac{E'}{E} = \beta S \frac{I}{E} - (\mu + \eta) \qquad (1.13)$$
$$\frac{I'}{I} = \eta \frac{E}{I} - (\mu + \gamma) + \alpha \frac{E}{I}. \qquad (1.14)$$

Putting (1.14) and (1.13) in (1.12) and (1.11) respectively, we get

$$k_1 = -(\beta I + \mu + \alpha) - (\mu + \eta) + \beta S \le \beta S \frac{I}{E} - (\mu + \eta)$$
$$k_2 = -2(\mu + \eta) - (\beta I + \mu + \alpha) + \beta S \frac{I}{E} + \frac{E}{I}(\beta S - \eta) - \alpha \frac{S}{I} \le \beta S \frac{I}{E} - (\mu + \eta).$$

Hence, from (1.10)

$$\varphi(B) \le \beta S \frac{I}{E} - (\mu + \eta) \text{ and so, } \frac{1}{t}\int_0^t \varphi(B) ds \le \frac{1}{t}\log_e(\beta S \frac{I}{E} - (\mu + \eta)).$$

So,$\bar{q}_2 < 0$, and hence the Bendixson criteria is also satisfied, which thus proves the global stability of the endemic equilibrium.

4. NUMERICAL SIMULATIONS AND EFFECT OF PARAMETERIC VALUES:
In this section, a large number of numerical simulations have been carried out to verify the analytical results of local and global stability obtained in section 3. Some examples of numerical simulations for $R_0 < 1$ and $R_0 > 1$ are mentioned below.
Example 1: The local stability of the rumor -free equilibrium point when $R_0 = \mathbf{0.5079}$ and $R_0 = \mathbf{0.0769}$has been numerically simulated and depicted in Figure1 and 2. It is clearly observed that the rumor-free equilibrium point turns out to be stable when $R_0 = \mathbf{0.5079} < 1$and $R_0 = \mathbf{0.0769} < 1.$
Example 2: The local stability of the endemic equilibrium point when $R_0 = \mathbf{1.3158}$ and $R_0 = \mathbf{1.2698}$has been numerically simulated and depicted in Figure 3 and 4. It is clearly observed that the endemic equilibrium point turns out to be stable when $R_0 = \mathbf{1.3158} > 1$ and $R_0 = \mathbf{1.2698} > 1$.
Example 3: The global stability of endemic equilibrium of system (1.1 to 1.4) for different gamma(γ) values i.e. 0.6, 0.7, 0.8, 0.9 and 1.0 the corresponding values of $\boldsymbol{R_0} = \mathbf{0.0769}, \boldsymbol{R_0} = \mathbf{0.0714}, \boldsymbol{R_0} = \mathbf{0.0667}, \boldsymbol{R_0} = \mathbf{0.0625}\ \boldsymbol{and}\ \boldsymbol{R_0} = \mathbf{0.0588} < 1$ has been numerically simulated and depicted in Figure 5 on the plane formed by the exposed population(E) and infectious population(I). From figure 5, it is observed that all resulting trajectories in the (E - I) plane are clockwise steady spiral and all trajectories converge to a unique equilibrium point$(E^*,\ I^*)$. This example also numerically shows that a unique endemic equilibrium point is globally asymptotically stable for$\boldsymbol{R_0} = \mathbf{0.0769}, \boldsymbol{R_0} = \mathbf{0.0714}, \boldsymbol{R_0} = \mathbf{0.0667}, \boldsymbol{R_0} = \mathbf{0.0625}\ \boldsymbol{and}\ \boldsymbol{R_0} = \mathbf{0.0588} < 1.$
Example 4: The global stability of endemic equilibrium of system (1.1 to 1.4) for different gamma(γ) values i.e. 0.6, 0.7, 0.8, 0.9 and 1.0 the corresponding values of $\boldsymbol{R_0} = \mathbf{0.0769}, \boldsymbol{R_0} = \mathbf{0.0714}, \boldsymbol{R_0} = \mathbf{0.0667}, \boldsymbol{R_0} = \mathbf{0.0625}\ \boldsymbol{and}\ \boldsymbol{R_0} = \mathbf{0.0588} < 1$has been numerically simulated and depicted in Figure 6 on the plane formed by the susceptible population(S) and infectious population(I). From figure 6, it is observed that all resulting trajectories in the (S - I) plane are anticlockwise steady spiral and all trajectories converge to a unique equilibrium point(S^*, I^*). This example also numerically shows that a unique endemic equilibrium point is globally asymptotically stable for $\boldsymbol{R_0} = \mathbf{0.0769}, \boldsymbol{R_0} = \mathbf{0.0714}, \boldsymbol{R_0} = \mathbf{0.0667}, \boldsymbol{R_0} = \mathbf{0.0625}\ \boldsymbol{and}\ \boldsymbol{R_0} = \mathbf{0.0588} < 1.$
Example 5: The behaviour of system (1.1 to1.4) is also studied for different values of R_0 by considering infected(I) population- recovered(R) population. Figure 7for different $\boldsymbol{R_0} = \mathbf{0.0769}, \boldsymbol{R_0} = \mathbf{0.0714}, \boldsymbol{R_0} = \mathbf{0.0667}, \boldsymbol{R_0} =$

$\mathbf{0.0625}$ ***and*** $\boldsymbol{R_0} = \mathbf{0.0588} < 1$respectively reflects that as the value of $gama(\gamma)$ increases for a constant value of $eta(\eta = 0.1)$, the recovered population also increases.

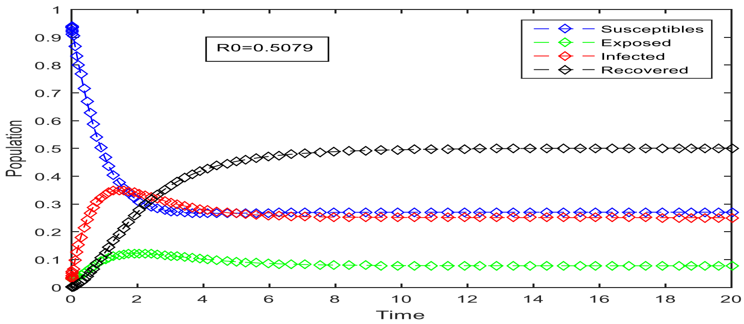

Figure 1. Time evolution of susceptible,exposed,infected and recovered population when β=0.8 mu=0.3,alpha=0.72,gamma=0.6 and eta=0.4 .

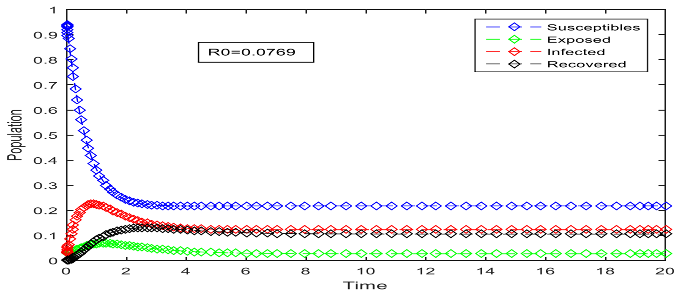

Figure 2. Time evolution of susceptible,exposed,infected and recovered population when β=0.8 mu=0.7,alpha=0.72,gamma=0.6 and eta=0.1.

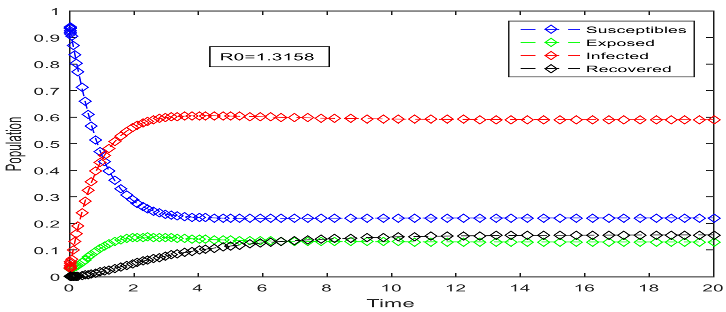

Figure 3. Time evolution of susceptible,exposed,infected and recovered population when β=0.8 mu=0.3,alpha=0.72,gamma=0.08 and eta=0.5 .

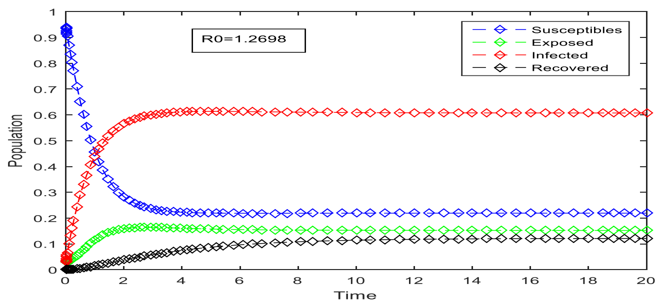

Figure 4. Time evolution of susceptible,exposed,infected and recovered population β=0.8 mu=0.3,alpha=0.72,gamma=0.06 and eta=0.4 .

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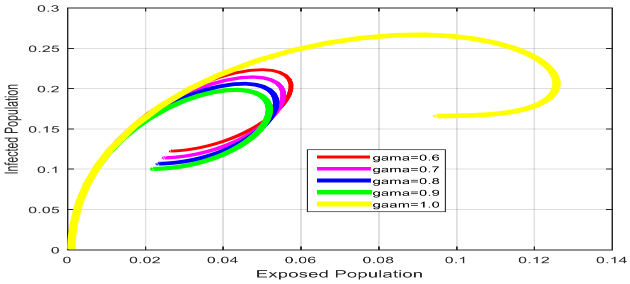

Figure 5: Infected versus Exposed population.

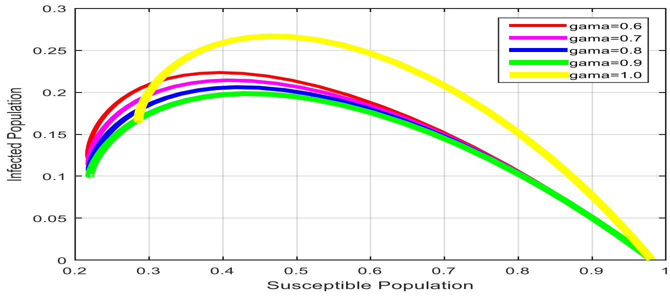

Figure 6: Susceptible verses Infected population.

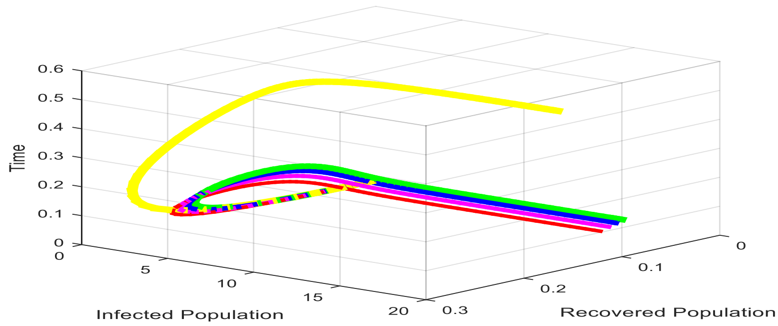

Figure 7: Recovered verses Infected population.

Parameter	Value	Source
μ	0.3 - 0.7	Assumed
γ	0.6-1.0	Assumed
β	0.8	[17]
η	0.1 - 0.5	Assumed
α	0.72	[18]

Table 2. Estimated values of the variables and parameters utilized in the model's simulation.

5. CONCLUSION:

A mathematical model called SEIR(Susceptible-Exposed-Infected-Recovered) is developedfor rumor transmission on social networks. The basic reproduction number for the model is obtained, and the conditions for local and global stability are well established.When the basic reproduction number is greater than one, the endemic equilibrium in the S-I phase plane of the model is globally asymptotically stable under different valuesof reproduction number.

The analysis carried out reflects that rumorhas a life span that depends on the type of spreader class [Figures 1-4] and the analysis also shows that as the value of $gamma(\gamma)$, increases for a constant value of $eta(\eta = 0.1)$, the recovered population also increases [Figure 7] which indicates that rumor has stopped spreading.

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