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A New Notion Of Open Sets In Nano Topological Spaces

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ABSTRACT

In this paper, we introduce a new class of nano sets referred to as nano b𝕔-open sets(briefly № b𝕔-open) in nano topological space (№TS).In addition, we investigate its fundamental properties.

Keywords: b-open sets, №-b-open sets,№- b𝕔 open sets, №- b𝕔 closed sets.
AMS Subject Classification: 54A05, 54A10

I. INTRODUCTION

The study of nano topology was started by M. Lellis Thivagar et al [8] with regard to a subset X of a universe that is described in terms of lower, upper and boundary approximations of X. He additionally described nano interior and nano closure in nano topological spaces.(or briefly №T Spaces). Andrijevic [1] presented and studied a category of generalized open sets in a topological space referred to as b-open sets. Further C. Indirani et al [4] created and studied nano b-open sets (№ bo sets) in nano topological spaces (№T Spaces). B𝕔 open sets were first introduced in topological spaces by Hariwan Z. Ibrahim [6]
In this study, a new class of open sets named asnano b𝕔- open sets in№TS is presented and its characteristics and properties are examined.

II. PRELIMINARIES

Definition 2.1. [8] Let U denote a non-empty finite set of elements referred to as universe and R represents an equivalence relation on Unamed as the indiscernibility relation. Elements belonging to the same equivalence class are saidto be indiscernible with one another. The pair (U, R) is called as the **approximation space.** Let X⊆ U.

(i) ***The lower approximation*** of X withrespect to R is the set of all elements, which canbe forcertain classified asXwith respect to Rand it is denoted by $L_{\mathcal{R}}(X)$.That is, $L_{\mathcal{R}}(X)=\cup_{X\in U}\{R_x : R_x \subseteq X\}$ where $R_x$denotes the equivalence class determined by x ∈U.

(ii) ***The upper approximation***of X with respect to R is the set of all elements, which can be possibly classified asX with respect to R and it is denoted by$U_{\mathcal{R}}(X)$. That is$U_{\mathcal{R}}(X) = \cup_{X\in U}\{ R_x: R_x \cap X\neq \emptyset \}$.

(iii) ***The boundary region*** of X with respect to R is the set of all elements, that can be classified neither as X nor as not-X with respect toR and it isdenoted by$B_{\mathcal{R}}(X)$. That is, $B_R(X)=U_{\mathcal{R}}(X)-L_{\mathcal{R}}(X)$.

Definition 2.2. [8] Let Urepresent the universe and R represent an equivalence relation on U. Then$\tau_R(X)$=$№^T$ = { U, ∅, $L_{\mathcal{R}}(X)$,$U_{\mathcal{R}}(X)$, $B_{\mathcal{R}}(X)$} where X⊆U.Then, $\tau_{\mathcal{R}}(X)$satisfies the axioms listed below.
(i) Uand ∅ ∈ $\tau_{\mathcal{R}}(X)$.
(ii) The union of the elements of any sub collection of $\tau_{\mathcal{R}}(X)$ is in $\tau_{\mathcal{R}}(X)$.
(iii) The intersection of the elements of any finite subcollection of $\tau_{\mathcal{R}}(X)$ is in $\tau_{\mathcal{R}}(X)$.That is, $\tau_{\mathcal{R}}(X)$is a topology on U referred to as the ***nano topology***($№^T$) on U with respect to X. We call (U,$\tau_{\mathcal{R}}(X)$) (or) (U,$№^T$)as the ***nano topological space***(№TS-in short).The elements of$№^T$ are known as № ***open sets***(briefly,№-OS). The complement of №-open sets are №-***closed***sets(briefly, №-CS).

Example 2.3. [8**]** Let U={p,q,r,s} with U/R = {{p},{q},{r,s}}and X={p,r} ⊂U. Then the nano topology is $\tau_{\mathcal{R}}$(X) =№^{T}={ U, ∅,{p},{r,s},{p,r,s}}.

Remark 2.4. [8] If $\tau_{\mathcal{R}}$(X) =№^{T}is the nano topology on Uwith respect to X and B_N isanano subset of №TS,then B_N ={ U, $L_{\mathcal{R}}$(X), $B_{\mathcal{R}}$(X)} is referred to as the basis for $\tau_{\mathcal{R}}$(X) .

Definition 2. [8] If (U,№^{T}) is a nano T. space (№TS) with respect to X where X⊆Uand ifA_N is a nano subset in №TS and if A_N⊆U, then

(1) The **Nano interior**of $A_N$is defined as the union of all nano-open subsets of A and it is denoted by №int(A_N). That is, №int(A_N)is the largest nano-open subset of A_N.

(2) The **Nano closure** of $A_N$is defined as the intersection of all nano closed sets containing $A_N$and it is denoted by №cl(A_N). That is, №cl(A_N) is the smallestnano closed set containing A_N.

Definition 2.6 . Let (U, $\tau_{\mathcal{R}}$(X)) be a №TS and A_N⊆U. Then A_N is said to be

(1) **Nano-semiopen**set (№-SO set) [8] if A_N⊆№cl[№int(A_N)] and **Nano semi-closed** (№-SC set) [7] if №int[№cl(A_N)]⊆A_N.

(2) **Nano-θ open**set(№-θO set) [3] if for each x∈A_N, there exists a nano open set (№-OS) G such that x ∈ G ⊂№cl(G)⊂A_N.

(3) **Nano-θsemiopen** (№-θSO) [3] if for each x ∈A_N ,there exists a nano semi open set (№-SO set) G such that x∈G⊂№cl(G)⊂A_N.

№-SO(U, X), №-θO(U, X) and №-θSO(U, X) respectively denote the families of all nano semi-open(№-SO), nano θ-open(№-θO) and nano θ semi-open(№-θSO)subsets of U.

Definition 2.7. [3**]** Let (U,$\tau_{\mathcal{R}}$(X)) is a №TS and A_N⊆U. ThenA_N is said to be nano- bopen set (№ bo set) ifA_N ⊆ №cl(№int(A_N)) ∪ №int(№cl(A_N)). The complement of nano- bopen set is called nano- bclosed set (№ bℂset).

Example 2.8. [3**]** Let U = {p,q,r,s} with U/R ={p}, {r}, {q,s} and X = {p,q}.
Then the nano topology τ_R(X)= №^{T}= { U, ∅, {p}, {p,q,s}, {q,s}} and nano b-open sets are U, ∅, {p}, {q}, {s},{p, q},{p,r},{p,s},{q, s},{p,q, r},{p,q, s},{q,r,s}.

Definition 2.9. [10**]** A №TS (U, №^{T}) is referred to as nano locally Indiscrete space if every nano open set (№-OS) is nano closed set.(№-CS).

III. NANO-Bℂ OPEN SETS (№-bℂ open)

Definition 3.1. A nano set A_N of a nano topological space ($\tilde{U}_N$, τ_N) is called nano bℂ - open set (№-bℂ- OS) if for every $\varkappa \in A_N \in$ №-BO($\tilde{U}_N$, τ_N), there exists a nano closed set (№-CS) $\mathcal{H}_N$ such that $\varkappa \in \mathcal{H}_N \subset A_N$.

The family of all nano bℂ-open sets of a Nano topological space (NTS in short) ($\tilde{U}_N$, τ_N) is denoted by №-BℂO($\tilde{U}_N$, τ_N).

Example 3.2. Let $\tilde{U}_N$ ={ⓞ$_1$, ⓞ$_2$, ⓞ$_3$, ⓞ$_4$},$\tilde{U}_N$ / $\mathcal{R}$ ={{ⓞ$_1$},{ⓞ$_3$},{ⓞ$_2$, ⓞ$_4$}} and X_N ={ⓞ$_1$, ⓞ$_2$} ⊂ $\tilde{U}_N$. Then the Nano topology №^{T} = $\tau_{\mathcal{R}}$(X_N)= {$\tilde{U}_N$, $\emptyset_N$, {ⓞ$_1$},{ⓞ$_1$, ⓞ$_2$, ⓞ$_4$},{ⓞ$_2$, ⓞ$_4$}}. Then the nano Closed sets are $\tilde{U}_N$, $\emptyset_N$, {ⓞ$_2$, ⓞ$_3$, ⓞ$_4$},{ⓞ$_3$} and {ⓞ$_1$, ⓞ$_3$}. Then the collection of all №-b-open sets are №-bO($\tilde{U}_N$, X_N) = {$\tilde{U}_N$, $\emptyset_N$, {ⓞ$_1$},{ⓞ$_2$},{ⓞ$_4$},{ⓞ$_1$, ⓞ$_2$},{ⓞ$_1$, ⓞ$_3$}, {ⓞ$_1$, ⓞ$_4$},{ⓞ$_2$, ⓞ$_4$},{ⓞ$_1$, ⓞ$_2$, ⓞ$_3$},{ⓞ$_1$, ⓞ$_2$, ⓞ$_4$},{ⓞ$_2$, ⓞ$_3$, ⓞ$_4$}} and №-bℂO($\tilde{U}_N$, X_N) = {$\tilde{U}_N$, $\emptyset_N$, {ⓞ$_2$, ⓞ$_3$, ⓞ$_4$},{ⓞ$_1$, ⓞ$_3$}}.

Theorem 3.3. A nano subset A_N of a NTS ($\tilde{U}_N$, τ_N) is №-bℂ-open (№-bℂ-OS) if and only if A_N is №-b-open and it is a union of №-closed sets (№-CS). That is $A_N = \cup \mathcal{F}_\alpha$, where A_N is №-b-open and $\mathcal{F}_\alpha$ is №-closed set for each α.

Proof: (⇒)
Let A_N be a №-bℂ-OS. Then A_N is №-b-open and for each $\varkappa \in A_N$, there is a №-CS $\mathcal{F}_N$ such that $\varkappa \in \mathcal{F}_N \subset A_N$. Then we get U $\{\varkappa\}_{x\in A}$ = $A_N \subset \mathcal{F}_N \subset A_N$. Thus $A_N = \cup \mathcal{F}_N$, where A_N is №- closed set for each $\varkappa \in A_N$.
(⇐) Direct from the definition of №-bℂ-open set .

Remark 3.4. Every №-bℂ-OS of a NTS ($\tilde{U}_N$, τ_N) is №-b-open, but the converse is not true in general as shown by the following example.

Example 3.5. In example 3.2 , {ⓞ$_1$} ∈ №-bO($\tilde{U}_N$, X_N) but {ⓞ$_1$}∉ №-bℂ-O($\tilde{U}_N$, X_N) Hence {ⓞ$_1$}is №-b-open but not №-bℂ-O set.

Theorem 3.6. An arbitrary union of №-b𝕔-OS in $(\tilde{U}_N, \tau_N)$ is №-b𝕔 open.

Proof: Suppose $\{A_w : w \in \Delta\}$ is a family of №-b𝕔-OS in NTS $(\tilde{U}_N, \tau_N)$.Then A_w is №-b-OS for every $w \in \Delta$. So, $\cup A_w$ is №-b-OS . Let $\varkappa \in \cup\{A_w : w \in \Delta\}$. So $\varkappa \in A_w$ for some $w \in \Delta$. Since A_w is№-bO for every w, there exist a №-CS $\mathcal{F}_N$ such that $\varkappa \in \mathcal{F}_N \subset A_w \subset \cup\{A_w : w \in \Delta\}$. So $\varkappa \in \mathcal{F}_N \subset \cup\{A_w : w \in \Delta\}$. Hence $\cup\{A_w : w \in \Delta\}$ is №-b𝕔-OS .

Remark 3.7. The following example shows that the intersection of two №-b𝕔-OS need not be №-b𝕔-OS in NTS $(\tilde{U}_N, \tau_N)$.

Example 3.8. Consider example 2.2.2, $\{\omega_1,\omega_3\} \in$ №-b𝕔-O$(\tilde{U}_N, \tau_N)$ and $\{\omega_2, \omega_3, \omega_4\}\in$ №-b𝕔-O$(\tilde{U}_N, \tau_N)$ but $\{\omega_1,\omega_3\} \cap \{\omega_2, \omega_3, \omega_4\} = \{\omega_3\} \notin$ №-b𝕔-O$(\tilde{U}_N, \tau_N)$.

Remark 3.9. From the above example, we note that the family of all №-b𝕔-open subsets of a NTS $(\tilde{U}_N, \tau_N)$ is not a nano topology in general.

The following result gives a condition under which the family of all №-b𝕔-open sets became a nano topology on $\tilde{U}_N$.

Theorem 3.10. If the collection of all №-b-open sets of a NTS $(\tilde{U}_N, \tau_N)$ is a nano topology(NT) on $(\tilde{U}_N, \tau_N)$,then the collection of №-b𝕔-O sets is also a NT on $(\tilde{U}_N, \tau_N)$.

Proof. Clearly φ_N and $\tilde{U}_N \in$№-b𝕔-O$(\tilde{U}_N, \tau_N)$ and by theorem 3.6, the union of any family of №-b𝕔-O sets is №-b𝕔-O. Now let A_N and B_N be two №-b𝕔-O sets. Then A_N and B_N are №-bO sets. Since №-bO$(\tilde{U}_N, X_N)$ is a nano topology on $(\tilde{U}_N, X_N)$,$A_N \cap B_N$ is №-bO. Let $X_N \in A_N \cap B_N$, then $X_N \in A_N$ and $X_N \in B_N$. So there exists two №-C sets F_N and E_N such that $X_N \in F_N \subset A_N$ and $X_N \in E_N \subset B_N$. This implies that $X_N \in F_N \cap E_N \subset A_N \cap B_N$. Since any intersection of №-C sets is №-C set,$F_N \cap E_N$ is №-C set. Thus $A_N \cap B_N$ is №-b𝕔-O set.

Theorem 3.11. Every №-θOS in $(\tilde{U}_N, \tau_N)$ is №-b𝕔-OS in $(\tilde{U}_N, \tau_N)$.

Proof . Let A_N be №-θOS in $(\tilde{U}_N, \tau_N)$.Then for each $X_N \in A_N$, there exist №-OS G_N such that $X_N \in G_N \subset$ №-cl $(G_N) \subset A_N$. So $\cup\{X_N\} \in \cup G_N \subset \cup$ №-cl $(G_N) \subset A_N$. Then $A_N = \cup G_N$ which is №- OS and $A_N = \cup$№-cl (G_N) is a union of №-C sets. By theorem 2.2.3, A_N is №-b𝕔-OS in $(\tilde{U}_N, \tau_N)$.

Remark 3.12. The converse of the above theorem need not be true.

Example 3.13. Let $\tilde{U}_N =\{\omega_1, \omega_2, \omega_3, \omega_4\}$,$\tilde{U}_N / \mathcal{R} =\{\{\omega_1\},\{\omega_3\},\{\omega_2, \omega_4\}\}$ and $X_N =\{\omega_1, \omega_2\} \subset \tilde{U}_N$. Then NT $\tau_{\mathcal{R}}(X_N) =\{\tilde{U}_N, \emptyset_N, \{\omega_1\},\{\omega_1, \omega_2, \omega_4\},\{\omega_2, \omega_4\}\}$. $\tau_{\mathcal{R}}{}^c =\{\tilde{U}_N, \emptyset_N, \{\omega_2, \omega_3, \omega_4\}, \{\omega_3\}$ and $\{\omega_1, \omega_3\}\}$.Consider $A_N = \{\omega_1,\omega_3\}$ which is№-b𝕔-OS but not №-θOS in $(\tilde{U}_N, \tau_N)$.

Theorem 3.14. Every №-θS-OS in $(\tilde{U}_N, \tau_N)$ is №-b𝕔-OS in $(\tilde{U}_N, \tau_N)$.

Proof. Let A_N be №-θS-OS in $(\tilde{U}_N, \tau_N)$.Let G_N be №-SO such that for each $X_N \in A_N$,,$X_N \in G_N \subset$ №-cl $(G_N) \subset A_N$. Then $A_N = \cup G_N$ and $A_N = \cup$№-cl (G_N) is a union of №-C sets Hence A_N is №-b𝕔-OS in $(\tilde{U}_N, \tau_N)$.

Remark 3.15. The converse of the above theorem need not be true.

Example 3.16. Let $\tilde{U}_N =\{\omega_1, \omega_2, \omega_3, \omega_4\}$,$\tilde{U}_N / \mathcal{R} =\{\{\omega_1, \omega_2\},\{\omega_3\},\{\omega_4\}\}$ and $X_N =\{\omega_2, \omega_3\} \subset \tilde{U}_N$. Then NT $\tau_{\mathcal{R}}(X_N) =\{\tilde{U}_N, \emptyset_N, \{\omega_3\},\{\omega_1,\omega_3,\omega_4\},\{\omega_1, \omega_4\}\}$. $\tau_{\mathcal{R}}{}^c =\{\tilde{U}_N, \emptyset_N,\{\omega_1,\omega_2,\omega_4\}, \{\omega_2\}$ and $\{\omega_2, \omega_3\}\}$.Consider $A_N = \{\omega_3\}$ which is№-b𝕔-OS but not №-θS-OS in $(\tilde{U}_N, \tau_N)$.

Theorem 3.17. Every №-b𝕔-OS in $(\tilde{U}_N, \tau_N)$ is №-β-OS in $(\tilde{U}_N, \tau_N)$.

Proof. Let A_N be №-b𝕔-OS in $(\tilde{U}_N, \tau_N)$. Then for any $X_N \in A_N \in$ №-bO$(\tilde{U}_N, X_N)$, there exist a №- CS $\mathcal{H}_N$ in $(\tilde{U}_N, \tau_N)$ such that $\varkappa \in \mathcal{H}_N \subset A_N$. Since A_N is №-b𝕔-OS, A_N is №-bo such that $A_N \subset$ №-cl (№-int (A_N)) $\cup$ №-int (№-cl (A_N)) .But A_N is union of nano C sets. Hence $A_N \subset$ №-cl (№-int (№-cl (A_N))). Hence A_N is №-β-OS in $(\tilde{U}_N, \tau_N)$.

Remark 3.18. The converse of the above theorem need not be true.

Example 3.19. Let $\tilde{U}_N =\{\omega_1, \omega_2, \omega_3, \omega_4\}$,$\tilde{U}_N / \mathcal{R} =\{\{\omega_1\},\{\omega_3\},\{\omega_2, \omega_4\}\}$ and $X_N =\{\omega_1, \omega_2\} \subset \tilde{U}_N$. Then NT $\tau_{\mathcal{R}}(X_N) =\{\tilde{U}_N, \emptyset_N, \{\omega_1\},\{\omega_1, \omega_2, \omega_4\},\{\omega_2, \omega_4\}\}$. $\tau_{\mathcal{R}}{}^c =\{\tilde{U}_N, \emptyset_N, \{\omega_2, \omega_3, \omega_4\}, \{\omega_3\}$ and $\{\omega_1, \omega_3\}\}$. Consider $A_N = \{\tilde{U}_N, \emptyset_N, \{\omega_1\},\{\omega_2\}\}$. Here A_N is №-β-OS but not №-b𝕔-OS in $(\tilde{U}_N, \tau_N)$.

Theorem 3.20. Every №-b𝕔-OS in $(\tilde{U}_N, \tau_N)$ is №-b-OS in $(\tilde{U}_N, \tau_N)$.

Proof. Let A_N be №-b𝕔-OS in $(\tilde{U}_N, \tau_N)$. By the definition of №-b𝕔-OS ,it is obvious that A_N is №-b-OS and it is a union of nano closed sets.

Remark 3.21. The following examples shows that №-b𝕔- open sets and №-pre open sets are independent in ($\tilde{U}_N$, τ_N).

Example 3.22. Let $\tilde{U}_N$ ={$ɷ_1$, $ɷ_2$, $ɷ_3$, $ɷ_4$},$\tilde{U}_N$ / $\mathcal{R}$ ={{$ɷ_1$},{$ɷ_3$},{$ɷ_2$, $ɷ_4$}} and X_N ={$ɷ_1$, $ɷ_2$} ⊂ $\tilde{U}_N$. Then NT $\tau_{\mathcal{R}}(X_N)$ ={$\tilde{U}_N$, $\emptyset_N$, {$ɷ_1$},{$ɷ_1$,$ɷ_2$,$ɷ_4$},{$ɷ_2$, $ɷ_4$}}. $\tau_{\mathcal{R}}{}^c$ ={$\tilde{U}_N$, $\emptyset_N$, {$ɷ_2$,$ɷ_3$,$ɷ_4$}, {$ɷ_3$} and {$ɷ_1$, $ɷ_3$}}.Consider A_N = { $\tilde{U}_N$, $\emptyset_N$, {$ɷ_1$, $ɷ_2$},{$ɷ_1$, $ɷ_4$}}. Here A_N is №-pre open set but not №-b𝕔-OS in ($\tilde{U}_N$, τ_N).

Example 3.23. Let $\tilde{U}_N$ ={$ɷ_1$, $ɷ_2$, $ɷ_3$, $ɷ_4$},$\tilde{U}_N$ / $\mathcal{R}$ ={{$ɷ_1$, $ɷ_2$},{$ɷ_3$},{$ɷ_4$}} and X_N ={$ɷ_2$, $ɷ_3$} ⊂ $\tilde{U}_N$. Then NT $\tau_{\mathcal{R}}(X_N)$ ={$\tilde{U}_N$, $\emptyset_N$, {$ɷ_3$},{$ɷ_1$,$ɷ_3$,$ɷ_4$},{$ɷ_1$,$ɷ_4$}}. $\tau_{\mathcal{R}}{}^c$ ={$\tilde{U}_N$, $\emptyset_N$, {$ɷ_1$,$ɷ_2$,$ɷ_4$}, {$ɷ_2$} and {$ɷ_2$, $ɷ_3$}}.Consider A_N = {{$ɷ_2$, $ɷ_3$}}. Here A_N is №-b𝕔-OS but not №-pre open in ($\tilde{U}_N$, τ_N).

Theorem 3.24. Every №-b𝕔-OS in ($\tilde{U}_N$, τ_N) is №-S-OS in ($\tilde{U}_N$, τ_N).

Proof . Let A_N be №-b𝕔-OS in ($\tilde{U}_N$, τ_N). Since every №-b𝕔-OS is №-b open , A_N is №-b- open. Therefore we have A_N ⊂ №-cl (№-int (A_N)) ∪ №-int (№-cl (A_N)) and A_N is union of nano C sets . Hence A_N ⊂ №-cl (№-int (A_N)). Hence A_N is №-semi-OS in ($\tilde{U}_N$, τ_N).

Remark 3.25. The converse of the above theorem need not be true.

Example 3.26. Let $\tilde{U}_N$ ={$ɷ_1$, $ɷ_2$, $ɷ_3$, $ɷ_4$},$\tilde{U}_N$ / $\mathcal{R}$ ={{$ɷ_1$, $ɷ_2$},{$ɷ_3$},{$ɷ_4$}} and X_N ={$ɷ_2$, $ɷ_3$} ⊂ $\tilde{U}_N$. Then NT $\tau_{\mathcal{R}}(X_N)$ ={$\tilde{U}_N$, $\emptyset_N$, {$ɷ_3$},{$ɷ_1$,$ɷ_3$,$ɷ_4$},{$ɷ_1$,$ɷ_4$}}. $\tau_{\mathcal{R}}{}^c$ ={$\tilde{U}_N$, $\emptyset_N$, {$ɷ_1$,$ɷ_2$,$ɷ_4$}, {$ɷ_2$} and {$ɷ_2$, $ɷ_3$}}.Then №-SO ($\tilde{U}_N$, X_N) ={$\tilde{U}_N$, $\emptyset_N$, {$ɷ_3$},{$ɷ_1$,$ɷ_4$},{$ɷ_2$,$ɷ_3$}}. Consider A_N ={{$ɷ_1$,$ɷ_4$}}. Here A_N is№-SO but not №-b𝕔-OS in ($\tilde{U}_N$, τ_N).

Remark 3.27. The following examples shows that №-b𝕔-open sets and №-regular open sets are independent in ($\tilde{U}_N$, τ_N).

Example 3.28. Let $\tilde{U}_N$ ={$ɷ_1$, $ɷ_2$, $ɷ_3$, $ɷ_4$},$\tilde{U}_N$ / $\mathcal{R}$ ={{$ɷ_1$},{$ɷ_2$, $ɷ_4$},{$ɷ_3$}} and X_N ={$ɷ_1$, $ɷ_2$} ⊂ $\tilde{U}_N$. Then NT $\tau_{\mathcal{R}}(X_N)$ ={$\tilde{U}_N$, $\emptyset_N$, {$ɷ_1$},{$ɷ_1$,$ɷ_2$,$ɷ_4$},{$ɷ_2$,$ɷ_4$}}. $\tau_{\mathcal{R}}{}^c$ ={$\tilde{U}_N$, $\emptyset_N$, {$ɷ_2$,$ɷ_3$,$ɷ_4$},{$ɷ_3$} and {$ɷ_1$, $ɷ_3$}}.Then №-RO ($\tilde{U}_N$, X_N) ={$\tilde{U}_N$, $\emptyset_N$, {$ɷ_1$},{$ɷ_2$,$ɷ_3$}}. Consider A_N ={{$ɷ_2$,$ɷ_3$}}. Here A_N is №-RO but not №-b𝕔-OS in ($\tilde{U}_N$, τ_N).

Example 3.29. Let $\tilde{U}_N$ ={$ɷ_1$, $ɷ_2$, $ɷ_3$, $ɷ_4$},$\tilde{U}_N$ / $\mathcal{R}$ ={{$ɷ_1$},{$ɷ_2$, $ɷ_3$},{$ɷ_4$}} and X_N ={$ɷ_1$, $ɷ_2$} ⊂ $\tilde{U}_N$. Then NT $\tau_{\mathcal{R}}(X_N)$ ={$\tilde{U}_N$, $\emptyset_N$, {$ɷ_1$},{$ɷ_1$,$ɷ_2$,$ɷ_3$},{$ɷ_2$,$ɷ_3$}}. $\tau_{\mathcal{R}}{}^c$ ={$\tilde{U}_N$, $\emptyset_N$, {$ɷ_2$,$ɷ_3$,$ɷ_4$},{$ɷ_4$} and {$ɷ_1$, $ɷ_4$}}.Then №-b𝕔-OS ($\tilde{U}_N$, X_N) ={$\tilde{U}_N$, $\emptyset_N$,{$ɷ_1$,$ɷ_3$},{$ɷ_2$,$ɷ_3$,$ɷ_4$}}. Consider A_N ={{$ɷ_1$,$ɷ_3$}}. Here A_N is №-b𝕔-OS but not №-RO in ($\tilde{U}_N$, τ_N).

Remark 3.30. The following examples shows that №-b𝕔-OS and №-α-OS are independent in ($\tilde{U}_N$, τ_N).

Example 3.31. Let $\tilde{U}_N$ ={$ɷ_1$, $ɷ_2$, $ɷ_3$, $ɷ_4$},$\tilde{U}_N$ / $\mathcal{R}$ ={{$ɷ_1$},{$ɷ_3$},{$ɷ_2$, $ɷ_4$}} and X_N ={$ɷ_1$, $ɷ_2$} ⊂ $\tilde{U}_N$. Then NT $\tau_{\mathcal{R}}(X_N)$ ={$\tilde{U}_N$, $\emptyset_N$, {$ɷ_1$},{$ɷ_1$,$ɷ_2$,$ɷ_4$},{$ɷ_2$, $ɷ_4$}}. $\tau_{\mathcal{R}}{}^c$ ={$\tilde{U}_N$, $\emptyset_N$, {$ɷ_2$,$ɷ_3$,$ɷ_4$},{$ɷ_3$} and {$ɷ_1$, $ɷ_3$}}.Then №-α-OS ($\tilde{U}_N$, X_N) = {$\tilde{U}_N$, $\emptyset_N$, {$ɷ_1$},{$ɷ_2$,$ɷ_4$},{$ɷ_1$,$ɷ_2$, $ɷ_4$}}. Consider A_N ={$ɷ_2$, $ɷ_4$}. Here A_N is №-α-open set but not №-b𝕔-OS in ($\tilde{U}_N$, τ_N).

Example 3.32. Let $\tilde{U}_N$ ={$ɷ_1$,$ɷ_2$,$ɷ_3$,$ɷ_4$},$\tilde{U}_N$ / $\mathcal{R}$ ={{$ɷ_1$},{$ɷ_4$},{$ɷ_2$,$ɷ_3$}} and X_N ={$ɷ_1$,$ɷ_4$} ⊂ $\tilde{U}_N$. Then NT $\tau_{\mathcal{R}}(X_N)$ ={$\tilde{U}_N$, $\emptyset_N$, {$ɷ_1$,$ɷ_4$}}. $\tau_{\mathcal{R}}{}^c$ ={$\tilde{U}_N$, $\emptyset_N$,{$ɷ_2$,$ɷ_3$}}.Consider A_N = {$ɷ_2$,$ɷ_4$}. Here A_N is №-b𝕔-OS but not №-α-open set in ($\tilde{U}_N$, τ_N).

Remark 3.33. The following examples shows that №-b𝕔-OS and №-OS are independent in ($\tilde{U}_N$, τ_N).

Example 3.34. Let $\tilde{U}_N$ ={$ɷ_1$,$ɷ_2$,$ɷ_3$,$ɷ_4$},$\tilde{U}_N$ / $\mathcal{R}$ ={{$ɷ_1$},{$ɷ_3$},{$ɷ_2$,$ɷ_4$}} and X_N ={$ɷ_1$,$ɷ_2$} ⊂ $\tilde{U}_N$. Then NT $\tau_{\mathcal{R}}(X_N)$ ={$\tilde{U}_N$, $\emptyset_N$, {$ɷ_1$},{$ɷ_1$,$ɷ_2$,$ɷ_4$},{$ɷ_2$,$ɷ_4$}}. $\tau_{\mathcal{R}}{}^c$ ={$\tilde{U}_N$, $\emptyset_N$,{$ɷ_2$,$ɷ_3$,$ɷ_4$},{$ɷ_3$} and {$ɷ_1$,$ɷ_3$}}. Consider A_N ={ $ɷ_1$,$ɷ_2$,$ɷ_4$}. Here A_N is №-open set but not №-b𝕔-OS in ($\tilde{U}_N$, τ_N).

Example 3.35. Let $\tilde{U}_N$ ={$ɷ_1$,$ɷ_2$,$ɷ_3$,$ɷ_4$},$\tilde{U}_N$ / $\mathcal{R}$ ={{$ɷ_1$},{$ɷ_4$},{$ɷ_2$,$ɷ_3$}} and X_N ={$ɷ_1$,$ɷ_2$} ⊂ $\tilde{U}_N$. Then NT $\tau_{\mathcal{R}}(X_N)$ ={$\tilde{U}_N$, $\emptyset_N$, {$ɷ_1$},{$ɷ_2$,$ɷ_3$},{$ɷ_1$,$ɷ_2$,$ɷ_3$}}. $\tau_{\mathcal{R}}{}^c$ ={$\tilde{U}_N$, $\emptyset_N$,{$ɷ_2$,$ɷ_3$,$ɷ_4$}}.Consider A_N = {$ɷ_1$,$ɷ_3$}. Here A_N is №-b𝕔-OS but not №-open set in ($\tilde{U}_N$, τ_N).

Remark 3.36. The following diagram shows the relationships of №-b𝕔- open sets with some other nano open sets discussed in this section.

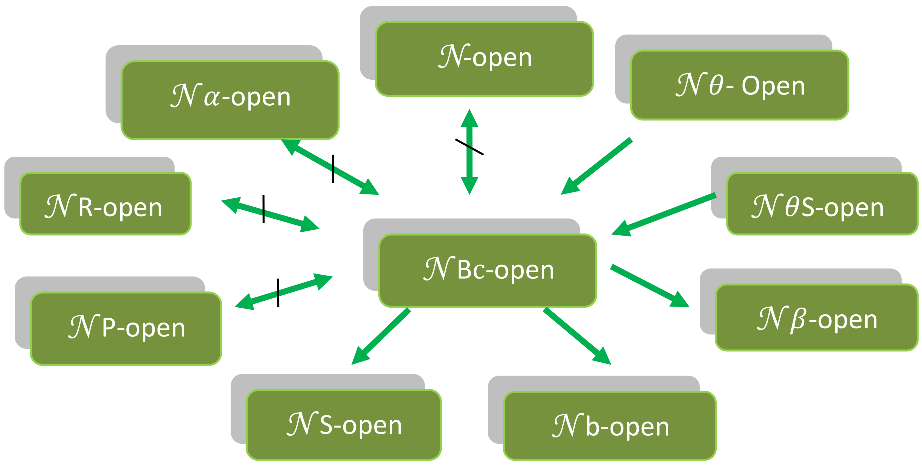

Figure 1 : Implications of №-bc- open set

where A $\longrightarrow$ B (resp.A $\longleftarrow\!\!\!|\!\!\!\longrightarrow$ B) represents A implies B (resp. A and B are independent).

IV Nano bc**-Closed Sets.**

Definition 4.1. A nano subset A_N of a nano topological space ($\tilde{U}_N$, τ_N) is called nano bc - closed set (№-bc-CS) if the complement of A_N is №-bc-open set in ($\tilde{U}_N$, τ_N). №-BcCS($\tilde{U}_N$, τ_N) is the collection of all №-bc-CS in ($\tilde{U}_N$, τ_N).

Example 4.2. Let $\tilde{U}_N$ ={ω_1, ω_2, ω_3, ω_4},$\tilde{U}_N$ / $\mathcal{R}$ ={{ω_1},{ω_3},{ω_2, ω_4}} and X_N ={ω_1, ω_2} ⊂ $\tilde{U}_N$. Then the Nano topology №^{T} = $\tau_{\mathcal{R}}(X_N)$= {$\tilde{U}_N$, $\emptyset_N$, {ω_1},{ω_1, ω_2, ω_4},{ω_2, ω_4}}. Then the nano Closed sets are $\tilde{U}_N$, $\emptyset_N$, {ω_2, ω_3, ω_4},{ω_3} and {ω_1, ω_3}. №-bO($\tilde{U}_N$, X_N) = {$\tilde{U}_N$, $\emptyset_N$, {ω_1}, {ω_2},{ω_4},{ω_1, ω_2},{ω_1, ω_3}, {ω_1, ω_4},{ω_2, ω_4},{ω_1, ω_2, ω_3},{ω_1, ω_2, ω_4},{ω_2, ω_3, ω_4}} and №-bcO($\tilde{U}_N$, X_N) = {$\tilde{U}_N$, $\emptyset_N$, {ω_2, ω_3, ω_4},{ω_1, ω_3}}. Consider A_N = {{ω_2, ω_3, ω_4}}which is №-bc-open set. The complement of A_N is №-bc-CS in ($\tilde{U}_N$, τ_N).

Theorem 4.3. A nano set A_N of a NTS ($\tilde{U}_N$, τ_N) is №-bc-closed (№-bc-CS) if and only if A_N is №-b-closed and it is an intersection of №-open sets (№-OS). That is A_N = ∩ $\mathcal{F}_\alpha$, where A_N is №-b-closed and $\mathcal{F}_\alpha$ is №-open set for each α.

Proof: (⇒)

Let A_N be a №-bc-CS. Then A_N is №-b-closed and A_N = №-cl (A_N) . Since A_N is №-b-closed , №-int (№-cl (A_N)) ∩ №-cl (№-int (A_N)) ⊂ A_N . For each $\varkappa \in A_N$, there is a №-OS $\mathcal{F}_N$ such that $\varkappa \in \mathcal{F}_N \subset A_N$. Thus A_N = ∩ $\mathcal{F}_N$.

(⇐) Direct from the definition of №-bc-closed set .

Theorem 4.4. Let{A_w : w ∈ Δ} be a collection of №-bc closed sets in ($\tilde{U}_N$, τ_N), then ∩{A_w : w ∈ Δ} is №-bc-CS.

Proof: Let A_w be a №-bc closed set for each w in $\tilde{U}_N$. This implies U/ A_w is №-bc-open in $\tilde{U}_N$. Then by theorem 3.6, U(U/ A_w) is №-bc-open in $\tilde{U}_N$. Then U/ ∩A_w is №-bc-open in $\tilde{U}_N$. Hence ∩A_w is №-bc closed in $\tilde{U}_N$.

Remark 4.5. The union of two №-bc closed sets in ($\tilde{U}_N$, τ_N) need not be №-bc-CS.

Example 4.6. Let $\tilde{U}_N$ ={ω_1, ω_2, ω_3, ω_4},$\tilde{U}_N$ / $\mathcal{R}$ ={{ω_1},{ω_3},{ω_2, ω_4}} and X_N ={ω_1, ω_2} ⊂ $\tilde{U}_N$. Then the Nano topology №^{T} = $\tau_{\mathcal{R}}(X_N)$= {$\tilde{U}_N$, $\emptyset_N$, {ω_1},{ω_1, ω_2, ω_4},{ω_2, ω_4}}. Then №-CS are $\tilde{U}_N$, $\emptyset_N$, {ω_2, ω_3, ω_4},{ω_3} and {ω_1, ω_3} №-bcC($\tilde{U}_N$, X_N) = {$\tilde{U}_N$, $\emptyset_N$, {ω_1},{ω_2, ω_4}}. Here {ω_1} ∈ №-bcC($\tilde{U}_N$, X_N) and {ω_2, ω_4}∈ №-bcC($\tilde{U}_N$, X_N). But{ω_1}∪{ω_2, ω_4}∉ №-bcC($\tilde{U}_N$, X_N).

Theorem 4.7. 1. Every №-θCS in ($\tilde{U}_N$, τ_N) is №-bc-CS in ($\tilde{U}_N$, τ_N).
2. Every №-θS-CS in ($\tilde{U}_N$, τ_N) is №-bc-CS in ($\tilde{U}_N$, τ_N).
3. Every №-bc-CS in ($\tilde{U}_N$, τ_N) is №-β-CS in ($\tilde{U}_N$, τ_N).
4. Every №-bc-CS in ($\tilde{U}_N$, τ_N) is №-S-CS in ($\tilde{U}_N$, τ_N).
5. Every №-bc-CS in ($\tilde{U}_N$, τ_N) is №-b-CS in ($\tilde{U}_N$, τ_N).
6. №-bc- closed sets and №-pre closed sets are independent in ($\tilde{U}_N$, τ_N).
7. №-bc-closed sets and №-regular closed sets are independent in ($\tilde{U}_N$, τ_N).
8. №-bc-CS and №-α-CS are independent in ($\tilde{U}_N$, τ_N).
9. №-bc-CS and №-CS are independent in ($\tilde{U}_N$, τ_N).

Proof : Straight forward.

Remark 4.8. The converse of the above theorem need not be true as seen in the following examples .

Example 4.9. Let $\tilde{U}_N = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, $\tilde{U}_N / \mathcal{R} = \{\{\omega_1\},\{\omega_3\},\{\omega_2, \omega_4\}\}$ and $X_N = \{\omega_1, \omega_2\} \subset \tilde{U}_N$. Then NT $\tau_{\mathcal{R}}(X_N) = \{\tilde{U}_N, \emptyset_N, \{\omega_1\},\{\omega_1, \omega_2, \omega_4\},\{\omega_2, \omega_4\}\}$. ${\tau_{\mathcal{R}}}^c = \{\tilde{U}_N, \emptyset_N, \{\omega_2, \omega_3, \omega_4\}, \{\omega_3\}$ and $\{\omega_1, \omega_3\}\}$.Consider $A_N = \{\omega_2,\omega_4\}$ which is№-b𝕔-CS but not №-θCS in $(\tilde{U}_N, \tau_N)$.

Example 4.10. Let $\tilde{U}_N = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, $\tilde{U}_N / \mathcal{R} = \{\{\omega_1, \omega_2\},\{\omega_3\},\{\omega_4\}\}$ and $X_N = \{\omega_2, \omega_3\} \subset \tilde{U}_N$. Then NT $\tau_{\mathcal{R}}(X_N) = \{\tilde{U}_N, \emptyset_N, \{\omega_3\},\{\omega_1,\omega_3,\omega_4\},\{\omega_1, \omega_4\}\}$. ${\tau_{\mathcal{R}}}^c = \{\tilde{U}_N, \emptyset_N, \{\omega_1,\omega_2,\omega_4\},\{\omega_2\}, \{\omega_2,\omega_3\}\}$.Consider $A_N = \{\omega_1,\omega_2,\omega_4\}$ which is№-b𝕔-CS but not №-θS-CS in $(\tilde{U}_N, \tau_N)$.

Example 4.11. Let $\tilde{U}_N = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, $\tilde{U}_N / \mathcal{R} = \{\{\omega_1\},\{\omega_3\},\{\omega_2, \omega_4\}\}$ and $X_N = \{\omega_1, \omega_2\} \subset \tilde{U}_N$. Then NT $\tau_{\mathcal{R}}(X_N) = \{\tilde{U}_N, \emptyset_N, \{\omega_1\},\{\omega_1, \omega_2, \omega_4\},\{\omega_2, \omega_4\}\}$. ${\tau_{\mathcal{R}}}^c = \{\tilde{U}_N, \emptyset_N, \{\omega_2, \omega_3, \omega_4\}, \{\omega_3\}$ and $\{\omega_1, \omega_3\}\}$.Consider $A_N = \{\tilde{U}_N, \emptyset_N, \{\omega_2, \omega_3, \omega_4\},\{\omega_1, \omega_2, \omega_4\}\}$. Here A_N is №-β-CS but not №-b𝕔-CS in $(\tilde{U}_N, \tau_N)$.

Example 4.12. Let $\tilde{U}_N = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, $\tilde{U}_N / \mathcal{R} = \{\{\omega_1, \omega_2\},\{\omega_3\},\{\omega_4\}\}$ and $X_N = \{\omega_2, \omega_3\} \subset \tilde{U}_N$. Then NT $\tau_{\mathcal{R}}(X_N) = \{\tilde{U}_N, \emptyset_N, \{\omega_3\},\{\omega_1,\omega_3,\omega_4\},\{\omega_1,\omega_4\}\}$. ${\tau_{\mathcal{R}}}^c = \{\tilde{U}_N, \emptyset_N, \{\omega_1,\omega_2,\omega_4\}, \{\omega_2\}$ and $\{\omega_2, \omega_3\}\}$.Then №-SCS$(\tilde{U}_N, X_N) = \{\tilde{U}_N, \emptyset_N, \{\omega_1, \omega_2,\omega_4\},\{\omega_2,\omega_3\},\{\omega_1,\omega_4\}\}$. Consider $A_N = \{\{\omega_1,\omega_4\}\}$. Here A_N is№-SCS but not №-b𝕔-CS in $(\tilde{U}_N, \tau_N)$.

Example 4.13. Let $\tilde{U}_N = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, $\tilde{U}_N / \mathcal{R} = \{\{\omega_1, \omega_2\},\{\omega_3\},\{\omega_4\}\}$ and $X_N = \{\omega_2, \omega_3\} \subset \tilde{U}_N$. Then NT $\tau_{\mathcal{R}}(X_N) = \{\tilde{U}_N, \emptyset_N, \{\omega_3\},\{\omega_1,\omega_3,\omega_4\},\{\omega_1,\omega_4\}\}$. ${\tau_{\mathcal{R}}}^c = \{\tilde{U}_N, \emptyset_N, \{\omega_1,\omega_2,\omega_4\}, \{\omega_2\}$ and $\{\omega_2, \omega_3\}\}$. Consider $A_N = \{\{\omega_1,\omega_4\}\}$. Here A_N is№-bCS but not №-b𝕔-CS in $(\tilde{U}_N, \tau_N)$.

Example 4.14. Let $\tilde{U}_N = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, $\tilde{U}_N / \mathcal{R} = \{\{\omega_1\},\{\omega_3\},\{\omega_2, \omega_4\}\}$ and $X_N = \{\omega_1, \omega_2\} \subset \tilde{U}_N$. Then NT $\tau_{\mathcal{R}}(X_N) = \{\tilde{U}_N, \emptyset_N, \{\omega_1\},\{\omega_1,\omega_2,\omega_4\},\{\omega_2, \omega_4\}\}$. ${\tau_{\mathcal{R}}}^c = \{\tilde{U}_N, \emptyset_N, \{\omega_2,\omega_3,\omega_4\}, \{\omega_3\}$ and $\{\omega_1, \omega_3\}\}$.Consider $A_N = \{\tilde{U}_N, \emptyset_N, \{\omega_3, \omega_4\},\{\omega_2, \omega_3\}\}$. Here A_N is №-pre closed set but not №-b𝕔-CS in $(\tilde{U}_N, \tau_N)$.

Example 4.15. Let $\tilde{U}_N = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, $\tilde{U}_N / \mathcal{R} = \{\{\omega_1, \omega_2\},\{\omega_3\},\{\omega_4\}\}$ and $X_N = \{\omega_2, \omega_3\} \subset \tilde{U}_N$. Then NT $\tau_{\mathcal{R}}(X_N) = \{\tilde{U}_N, \emptyset_N, \{\omega_3\},\{\omega_1,\omega_3,\omega_4\},\{\omega_1,\omega_4\}\}$. ${\tau_{\mathcal{R}}}^c = \{\tilde{U}_N, \emptyset_N, \{\omega_1,\omega_2,\omega_4\}, \{\omega_2\}$ and $\{\omega_2, \omega_3\}\}$.Consider $A_N = \{\{\omega_1, \omega_4\}\}$. Here A_N is №-b𝕔-CS but not №-pre closed set.

Example 4.16. Let $\tilde{U}_N = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, $\tilde{U}_N / \mathcal{R} = \{\{\omega_1\},\{\omega_2, \omega_4\},\{\omega_3\}\}$ and $X_N = \{\omega_1, \omega_2\} \subset \tilde{U}_N$. Then NT $\tau_{\mathcal{R}}(X_N) = \{\tilde{U}_N, \emptyset_N, \{\omega_1\},\{\omega_1,\omega_2,\omega_4\},\{\omega_2,\omega_4\}\}$. ${\tau_{\mathcal{R}}}^c = \{\tilde{U}_N, \emptyset_N, \{\omega_2,\omega_3,\omega_4\},\{\omega_3\}$ and $\{\omega_1, \omega_3\}\}$.Then №-RCS$(\tilde{U}_N, X_N) = \{\tilde{U}_N, \emptyset_N, \{\omega_2, \omega_3,\omega_4\},\{\omega_1,\omega_4\}\}$. Consider $A_N = \{\{\omega_1,\omega_4\}\}$. Here A_N is №-RCS but not №-b𝕔-CS in $(\tilde{U}_N, \tau_N)$.

Example 4.17. Let $\tilde{U}_N = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, $\tilde{U}_N / \mathcal{R} = \{\{\omega_1\},\{\omega_2, \omega_3\},\{\omega_4\}\}$ and $X_N = \{\omega_1, \omega_2\} \subset \tilde{U}_N$. Then NT $\tau_{\mathcal{R}}(X_N) = \{\tilde{U}_N, \emptyset_N, \{\omega_1\},\{\omega_1,\omega_2,\omega_3\},\{\omega_2,\omega_3\}\}$. ${\tau_{\mathcal{R}}}^c = \{\tilde{U}_N, \emptyset_N, \{\omega_2,\omega_3,\omega_4\},\{\omega_4\}$ and $\{\omega_1, \omega_4\}\}$.Then №-b𝕔-CS $(\tilde{U}_N, X_N) = \{\tilde{U}_N, \emptyset_N,\{\omega_2,\omega_4\},\{\omega_1\}\}$. Consider $A_N = \{\{\omega_2,\omega_4\}\}$. Here A_N is №-b𝕔-CS but not №-RCS in $(\tilde{U}_N, \tau_N)$.

Example 4.18. Let $\tilde{U}_N = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, $\tilde{U}_N / \mathcal{R} = \{\{\omega_1\},\{\omega_3\},\{\omega_2, \omega_4\}\}$ and $X_N = \{\omega_1, \omega_2\} \subset \tilde{U}_N$. Then NT $\tau_{\mathcal{R}}(X_N) = \{\tilde{U}_N, \emptyset_N, \{\omega_1\},\{\omega_1,\omega_2,\omega_4\},\{\omega_2, \omega_4\}\}$. ${\tau_{\mathcal{R}}}^c = \{\tilde{U}_N, \emptyset_N, \{\omega_2,\omega_3,\omega_4\},\{\omega_3\}$ and $\{\omega_1, \omega_3\}\}$.Then №-α-CS $(\tilde{U}_N, X_N) = \{\tilde{U}_N, \emptyset_N, \{\omega_2, \omega_3, \omega_4\},\{\omega_1,\omega_3\},\{\omega_3\}\}$. Consider $A_N =$
$\{\omega_1, \omega_3\}$. Here A_N is №-α-closed set but not №-b𝕔-CS in $(\tilde{U}_N, \tau_N)$.

Example 4.19. Let $\tilde{U}_N = \{\omega_1,\omega_2,\omega_3,\omega_4\}$, $\tilde{U}_N / \mathcal{R} = \{\{\omega_1\},\{\omega_4\},\{\omega_2,\omega_3\}\}$ and $X_N = \{\omega_1,\omega_4\} \subset \tilde{U}_N$. Then NT $\tau_{\mathcal{R}}(X_N) = \{\tilde{U}_N, \emptyset_N, \{\omega_1,\omega_4\}\}$. ${\tau_{\mathcal{R}}}^c = \{\tilde{U}_N, \emptyset_N, \{\omega_2,\omega_3\}\}$.Consider $A_N = \{\omega_1,\omega_3\}$. Here A_N is №-b𝕔-CS but not №-α-closed set in $(\tilde{U}_N, \tau_N)$.

Example 4.20. Let $\tilde{U}_N = \{\omega_1,\omega_2,\omega_3,\omega_4\}$, $\tilde{U}_N / \mathcal{R} = \{\{\omega_1\},\{\omega_3\},\{\omega_2,\omega_4\}\}$ and $X_N = \{\omega_1,\omega_2\} \subset \tilde{U}_N$ .Then NT $\tau_{\mathcal{R}}(X_N) = \{\tilde{U}_N, \emptyset_N, \{\omega_1\},\{\omega_1,\omega_2,\omega_4\},\{\omega_2,\omega_4\}\}$. ${\tau_{\mathcal{R}}}^c = \{\tilde{U}_N, \emptyset_N, \{\omega_2,\omega_3,\omega_4\},\{\omega_3\}$ and $\{\omega_1,\omega_3\}\}$. Consider $A_N = \{\omega_3\}$. Here A_N is №-closed set but not №-b𝕔-CS in $(\tilde{U}_N, \tau_N)$.

Example 4.21. Let $\tilde{U}_N = \{\omega_1,\omega_2,\omega_3,\omega_4\}$, $\tilde{U}_N / \mathcal{R} = \{\{\omega_1\},\{\omega_4\},\{\omega_2,\omega_3\}\}$ and $X_N = \{\omega_1,\omega_2\} \subset \tilde{U}_N$ .Then NT $\tau_{\mathcal{R}}(X_N) = \{\tilde{U}_N, \emptyset_N, \{\omega_1\},\{\omega_2,\omega_3\},\{\omega_1,\omega_2,\omega_3\}\}$. ${\tau_{\mathcal{R}}}^c = \{\tilde{U}_N, \emptyset_N, \{\omega_2,\omega_3,\omega_4\}\}$.Consider $A_N = \{\omega_2,\omega_3\}$. Here A_N is №-b𝕔-CS but not №-closed set in $(\tilde{U}_N, \tau_N)$.

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