On Paired Double Domination for Degree Splitting Graphs of Some Graphs

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Abstract:

In this paper, we introduced the new concept Paired double domination number for degree splitting graph of standard graphs. A paired – double dominating set of a graph G with no isolated vertex is a double dominating sets of vertices whose induced subgraph has a perfect matching. A Paired double domination number $\gamma_{prdd}(G)$ is the minimum cardinality of a paired double dominating set of G. Let $G = (V, E)$ be a graph with $V(G) = S_1 \cup S_2 \cup ... \cup S_t \cup T$, where S_i is the set having at least two vertices of same degree and $T = V(G) - \cup S_i$, where $1 \leq i \leq t$. The degree splitting graph DS(G) is obtained from G by adding vertices $w_1, w_2, ... w_t$ and joining w_i to each vertex of S_i for $i = 1,2, ... t$. We establish Paired double domination number of graphs path, cycle, star, bistar, complete bipartite and complete graph.
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Key words: Double domination number, Paired - double domination number, Locating paired - double domination number.

1   INTRODUCTION

Let $G = (V, E)$ be graph with vertex set V and edge set E. We obtain with some terminology. For a vertex v of a graph G, the open neighborhood of a vertex $v \epsilon V$ is $N(u) = \{ v/uv \in E \}$ and closed neighborhood of vertex $N[v] = N(v) \cup \{v\}$.

A subset $S \subseteq V$ is a dominating set of G, if for every vertex $v \epsilon V, |N[v] \cap S| \geq 1$. The domination number is the minimum cardinality of a dominating set of G. A subset S of V is double dominating set of G if for every vertex $v \epsilon V, |N[v] \cap s| \geq 2$, that is v is in S and has at least one neighbor in S and v is in V-S has at least two neighbors in S [4].

A set S is called paired – dominating set if it dominates V and $< S >$ contains   at least one perfect matching. A paired – dominating set S with matching M is a dominating set $S = \{v_1, v_2, v_3, ... , v_{2t-1}v_{2t}\}$ with independent edge set $M = \{e_1, e_2, ... , e_t\}$ where each edge e_j joins two elements of S, that is M is perfect matching of $< S >$. If $v_j v_k = e_i \in M$, we say that v_j and v_k are paired in S [5]. A set S is called a paired – double dominating set if it is a double dominating set and $< S >$ contains at least one perfect matching. The double domination number $\gamma_{dd}(G)$ is the minimum cardinality of double dominating set of G, the paired –domination number $\gamma_{pr}(G)$ is the minimum cardinality of paired dominating set of G and paired double domination number $\gamma_{prdd}(G)$ is the minimum cardinality of a paired double dominating set of G.

In [7], R. Ponraj and S. Somasundaram have initiated a study on degree splitting graph $DS(G)$ of a graph G which is defined as follows:

Let $G = (V, E)$ be a graph with $V(G) = S_1 \cup S_2 \cup ... \cup S_t \cup T$, where S_i is the set having at least two vertices of same degree and $T = V(G) - \cup S_i$, where $1 \leq i \leq t$. The degree splitting graph DS(G) is obtained from G by adding vertices $w_1, w_2, ... w_t$ and joining w_i to each vertex of S_i for $i = 1,2, ... t$.

Theorem 1.1. [8] For any path P_n , $\gamma_{prdd}(P_n) = \begin{cases} 2 & if\ n = 2 \\ does\ not\ exist & if\ n = 3 \\ 2\left\lfloor \frac{n}{3} \right\rfloor + 2 & other\ wise \end{cases}$

Theorem 1.2. [8] For any cycle C_n $\gamma_{prdd}(C_n) = 2\left\lceil \frac{n}{3} \right\rceil$

Theorem 1.3. [8] For any path P_n , n≠3 , $\gamma_{dd}(P_n)$ $\leq \gamma_{prdd}(P_n)$

Theorem 1.4. [8] For any cycle C_n , $\gamma_{dd}(C_n)$ $\leq \gamma_{prdd}(C_n)$

Theorem 1.5. [8] If n = 3k+2 where k ϵ N, then $\gamma_{prdd}(P_n) = \gamma_{prdd}(C_n)$.

2. MAIN RESULTS

In this Paper, we find Paired double domination number of degree splitting graph $DS(G)$ of the graphs path, cycle, star, bistar, complete bipartite and complete graph.

Theorem 2.1

For any integer $n \geq 3$, $\gamma_{prdd}\big(DS(P_n)\big) = 2\left\lfloor \frac{n}{3} \right\rfloor + 2$.

Proof:

Let $v_1, v_2, v_3, \dots v_{n-1}, v_n$ be the path P_n with partitions $S_1 = \{ v_2, v_3, \dots v_{n-1}\}$ and $S_2 = \{ v_1, v_n\}$. H be the γ_{prdd} set of $DS(P_n)$. To obtain $DS(P_3)$ from P_3 we add x which corresponds to S_2 also P_3 is isomorphic to C_4 and to obtain $DS(P_n)$ for $n \geq 4$. We add x_1 and x_2 which correspond to S_1 and S_2 respectively. As a result $V\big(DS(P_3)\big) = \{x, v_1, v_2, v_3\}$ and $V\big(DS(P_n)\big) = \{x_1, x_2, v_1, v_2, v_3, \dots v_{n-1}, v_n \}$ where $\left|V\big(DS(P_n)\big)\right| = n + 2$ for $n \geq 4$. Let $H_1 = \{v_i, i \equiv 1,2 \ (mod3)\}$ and $H_2 = \{v_n, x_1\}$.

Case(i) $n \equiv 0 \ (mod3)$

Then $H = H_1 \cup H_2$ Thus $< H >$ contains a P_6 graph and $\left(\frac{n}{3} - 2\right) P_2$ graph. Hence $\gamma_{prdd}\big(DS(P_n)\big) = 6 + 2\left(\frac{n}{3} - 2\right) = 2\left\lfloor \frac{n}{3} \right\rfloor + 2$.

Case(ii) $n \equiv 1 \ (mod3)$

Then $H = H_1 \cup H_2$. Thus $< H >$ contains a P_4 graph and $\left\lfloor \frac{n-2}{3} \right\rfloor P_2$ graph. Hence $\gamma_{prdd}\big(DS(P_n)\big) = 4 + 2\left\lfloor \frac{n-2}{3} \right\rfloor = 4 + 2\left\lfloor \frac{3k+1-2}{3} \right\rfloor = 4 + 2\left\lfloor \frac{3k-1}{3} \right\rfloor = 2k + 2 = 2\left(\frac{n-1}{3}\right) + 2 = 2\left\lfloor \frac{n}{3} \right\rfloor + 2$.

Case(ii) $n \equiv 2 \ (mod3)$

Then $H = H_1$. Thus $< H >$ contains $\left\lfloor \frac{n}{3} \right\rfloor$ P_2 graph. Hence $\gamma_{prdd}\big(DS(P_n)\big) = 2\left\lfloor \frac{n}{3} \right\rfloor = 2\left\lceil \frac{3k+2}{3} \right\rceil = 2k + 2 = 2\left(\frac{n-2}{3}\right) + 2 = 2\left\lfloor \frac{n}{3} \right\rfloor + 2$.

Theorem 2.2

For any integer $n \geq 3$, $\gamma_{prdd}\big(DS(C_n)\big) = 2\left\lceil \frac{n}{3} \right\rceil$.

Proof:

Let $v_1, v_2, v_3, \dots v_{n-1}, v_n$ be the cycle C_n and H be the γ_{prdd} set of $DS(C_n)$.

To obtain $DS(C_n)$ for $n \geq 3$, we add a vertex x which is adjacent to every vertex in C_n. As result $V\big(DS(C_n)\big) = \{x, v_1, v_2, v_3, \dots v_{n-1}, v_n \}$, where $\left|V\big(DS(C_n)\big)\right| = n + 1$. Clearly $DS(C_n)$ is isomorphic to the wheel graph W_n. Let $H_1 = \{v_i, i \equiv 1,2 \ (mod3)\}$ and $H_2 = \{v_{n-1}\}$.

Case(i) $n \equiv 0 \ (mod3)$

Then $H = H_1$. Thus $< H >$ contains $\left(\frac{n}{3}\right) P_2$ graph. Hence $\gamma_{prdd}\big(DS(C_n)\big) = 2\left(\frac{n}{3}\right) = 2\left\lceil \frac{n}{3} \right\rceil$.

Case(ii) $n \equiv 1 \ (mod3)$

Then $H = H_1$. Thus $< H >$ contains a P_4 graph and $\left(\left\lceil \frac{n}{3} \right\rceil - 1\right) P_2$ graph. Hence $\gamma_{prdd}\big(DS(C_n)\big) = 4 + 2\left(\left\lceil \frac{n}{3} \right\rceil - 1\right) = 4 + 2\left(\left\lceil \frac{3k+1}{3} \right\rceil - 1\right) = 2k + 2 = 2\left(\frac{n-1+3}{3}\right) = 2\left(\frac{n+2}{3}\right) = 2\left\lceil \frac{n}{3} \right\rceil$.

Case(ii) $n \equiv 2 \ (mod3)$

Then $H = H_1 \cup H_2$. Thus $< H >$ contains a P_6 graph and $\left(\left\lceil \frac{n}{3} \right\rceil - 2\right) P_2$ graph. Hence $\gamma_{prdd}\big(DS(C_n)\big) = 6 + 2\left(\left\lceil \frac{n}{3} \right\rceil - 2\right) = 6 + 2\left(\left\lceil \frac{3k+2}{3} \right\rceil - 2\right) = 2k + 2 = 2\left(\frac{n-2+3}{3}\right) = 2\left(\frac{n+1}{3}\right) = 2\left\lceil \frac{n}{3} \right\rceil$.

Theorem 2.3

For any integer $n \geq 2$, $\gamma_{prdd}\big(DS(K_{1,n})\big) = 4$.

Proof:

Let $v_1, v_2, v_3, \ldots v_{n-1}, v_n$ be the end vertices and v be the full vertex of the star $K_{1,n}$ and x be the corresponding vertex which is added to obtain the graph $DS(K_{1,n})$. Then $V\left(DS(K_{1,n})\right) = \{v, v_1, v_2, v_3, \ldots v_{n-1}, v_n, x\}$. Clearly $\left|V\left(DS(K_{1,n})\right)\right| = n + 2$.

Let H be the γ_{prdd} set of $DS(K_{1,n})$. Then $H = \{v_1, v_n\, v, x\}$. Then $<H>$ has a Perfect Matching. Hence $\gamma_{prdd}\left(DS(K_{1,n})\right) = 4$.

Theorem 2.4

For any bistar graph $B_{m,n}$ $\gamma_{prdd}\left(DS(B_{m,n})\right) = 4$.

Proof:

Consider the bistar graph $B_{m,n}$ with $V(B_{m,n}) = \{u, v, u_i, v_j/1 \le i \le m, 1 \le j \le n\}$. Here u_i and v_j are the vertices adjacent with u and v respectively. Let x_1 and x_2 be the corresponding vertices which are added to obtain $DS(B_{m,n})$. Then $V\left(DS(B_{m,n})\right) = \{u, v, u_i, v_j, x_1, x_2/1 \le i \le m, 1 \le j \le n\}$ and so $\left|V\left(DS(B_{m,n})\right)\right| = m + n + 4$. Let H be the γ_{prdd} set of $DS(B_{m,n})$. Then $H = \{, u, v, x_2, v_n\}$. Then $<H>$ has a Perfect Matching. Hence $\gamma_{prdd}\left(DS(B_{m,n})\right) = 4$.

Theorem 2.5

For any integer $m, n \ge 2$, $\gamma_{prdd}\left(DS(K_{m,n})\right) = 4$.

Proof:

Consider $K_{m,n}$ with $V(K_{m,n}) = \{u_i, v_j/1 \le i \le m, 1 \le j \le n\}$ with partition $V_1 = \{v_1, v_2, v_3, \ldots v_{m-1}, v_m\}$ and $V_2 = \{v_1, v_2, u_3, \ldots u_{n-1}, u_n\}$. Now we consider the following two cases.

Case (i) $m = n$.

In this case each vertex is of same degree and so let x be the added vertex which is adjacent to every u_i and v_j, $1 \le i \le m$ and $1 \le j \le n$. Thus, we obtain the graph $DS(K_{m,n})$. Then $V\left(DS(K_{m,n})\right) = \{u_i, v_j, x/1 \le i \le m, 1 \le j \le n\}$ and so $\left|V\left(DS(K_{m,n})\right)\right| = m + n + 1$. Let H be the γ_{prdd} set of $DS(K_{m,n})$. Then $H = \{v_1, v_2, u_1, u_2\}$. Then $\gamma_{prdd}\left(DS(K_{m,n})\right) = 4$.

Case (i) $m \ne n$.

In this case each vertex u_i is of same degree and each vertex v_j is of same degree where $deg(u_i) \ne deg(v_j)$ $1 \le i \le m$ and $1 \le j \le n$, so let x_1 and x_2 be added vertices where x_1 is adjacent to every u_i and x_2 is adjacent to every v_j. Thus, we obtain the graph $DS(K_{m,n})$. Then $V\left(DS(K_{m,n})\right) = \{u_i, v_j, x_1, x_2/1 \le i \le m, 1 \le j \le n\}$ and so $\left|V\left(DS(K_{m,n})\right)\right| = m + n + 2$. Let H be the γ_{prdd} set of $DS(K_{m,n})$. Then $H = \{v_1, u_1, x_1, x_2\}$. Then $\gamma_{prdd}\left(DS(K_{m,n})\right) = 4$.

Theorem 2.6

For any integer $n \ge 2$, $\gamma_{prdd}(DS(K_n)) = 2$.

Proof:

Each vertex is of same degree and so let x be the added vertex which is adjacent every u_i where $1 \le i \le n$. Thus, we obtain the graph $DS(K_n)$. Then $V(DS(K_n)) = \{u_i, , x/1 \le j \le n\}$ and so $\left|V(DS(K_n))\right| = n + 1$. Clearly $DS(K_n)$ is isomorphic to K_{n+1}. Let H be the γ_{prdd} set of $DS(K_n)$. Then $H = \{u_1, x\}$. Then $\gamma_{prdd}(DS(K_n)) = 2$.

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