Why am I confused? Commognitive Conflict in Non-ordinary Question About Number Division

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Abstract:

In the 21st century, learning requires students' creativity in developing questions to understand more of the concepts they had. The ordinary questions are expected to encourage students' skills and creativity to solve unusual questions. Meanwhile, the non-ordinary questions cause cognitive conflict and trigger commognitive conflict in students. Material division of integers can be used to investigate the source of these conflicts. By employing descriptive qualitative research, three numbers of questions consisting of two ordinary questions and one non-ordinary question wasakedto 40 participants, students of Mathematics Education from several universities in Indonesia. Two research subjects were selected based on the results of their works which indicated a commognitive conflict. The results of their works were then clarified by usingsemi-structured interviews to dig deeper into the causes of conflict in the components of words use, visual mediators, narratives, and routines. Confusion and doubts about their answers lead to commognitive conflict. The conflict occurs in the remaining negative quotient in terms of the word use. The subjects' visual mediators describe the commognitive conflicts that occur. Meanwhile in narratives and routines, commognitive conflict occurs sincethe subject's comprehendingare low regarding the remainder of the division which must be non-negative, leading to a commognitive conflict. Therefore, increasing the students' understanding of the concept of dividing integers is needed to reduce these conflicts.

Keywords: non-ordinary question, number division, commognitive, commognitive conflict

INTRODUCTION

In the 21st century learning requires students to have a variety of skills(Goodnough, Azam, & Wells, 2019). To face the all-digital millennial era, they must be able to develop their creativities and thinking skills, especially when they solve math problems(Aizikovitsh-Udi & Cheng, 2015; Chikiwa & Schäfer, 2018; Firdaus, Kailani, Bakar, & Bakry, 2015; Mulnix, 2012). Skills for creative thinking are the development of cognitive psychology and are the goals of education in all countries in the world(Hadinugrahaningsih, Rahmawati, & Ridwan, 2017; Jonsson, Norqvist, Liljekvist, & Lithner, 2014; Sitorus & Masrayati, 2016). Therefore, students are expected to be skilled and creative in solving math problems. Their abilities to comprehend a problem, develop a solution strategy, and carry out the problem-solving steps arecrucial to improve. The existence of a gap between the knowledge they acquired when they were in high school and university demanded them to be more creative in solving math problems(Bartelet, Ansari, Vaessen, & Blomert, 2014; Cirino, Fuchs, Elias, Powell, & Schumacher, 2015; Lai, Zhu, Chen, & Li, 2015; Rahim, Noor, & Zaid, 2015; Ramsburg & Ohlsson, 2016; Widodo & Wahyudin, 2018). Mathematics problems given in high school resulted a convergent way of thinking, unlikethose givenin universities which develop the principle of freedom and creative thinking in eachstudent to solve the problems(Kim et al., 2019a; Lu, Tao, Xu, & Stephens, 2020; Pratiwi, Nusantara, Susiswo, & Muksar, 2020; Thoma & Nardi, 2017, 2018). The limit of teacher's creativity in making math problems in high schools also causing limited ways of thinking.

In several previous studies, the learning process in schools can be optimized by building intensive communication between teachers and students(Ioannou, 2018; Kim et al., 2019b; Nardi, Ryve, Stadler, & Viirman, 2014; Sabouri, 2020; Viirman & Nardi, 2019). Students spend a lot of time at school communicating with teachers to improve their competences through some learning activities. Moreover, their creativitiescan be

built through problem-solving given to them by the teacher. The questions which are tiered from basic to advanced levels, which are expected to increase their creativities in learning, (León, Núñez, & Liew, 2014; Panadero & Romero, 2014; Schoenfeld, 2016) are designed to be given during the learning activities, either at the beginning, middle, or end.Students can verify the accuracy of their answers to the teacher or look for them on the internet. The questions given still refer to the concepts given by the teacher to students meaning there is no development of questions that trigger an increase in students' creativities. At universities, questions are given to students to develop further hence they become more creative and critical(Atabaki, Keshtiaray, & Yarmohammadian, 2015; Erikson & Erikson, 2019; Guerra & Holgaard, 2016; Widana, 2018). Creativity in formulatingquestions is closely related to thinking skills and the learning process(E.P.L. Emanuel, Kirana, & Chamidah, 2021; Endrayana Putut Laksminto Emanuel & Meilantifa, 2022). Problems that are not commonlyfound in learning activities are expected to improve students' creative skills in solving problems and their understanding of the materials provided(T. Jones, 2017; Larsson, 2017; Mumford, Todd, Higgs, & McIntosh, 2017; Palinussa, 2013; Yazar Soyadi, 2015). The increase of creative skills in problem-solving is an indication of an increase in their critical thinking skills.

Critical thinking skills are needed in learning to spur the ability in understanding the materials received (Aizikovitsh-Udi & Cheng, 2015; Chikiwa & Schäfer, 2018; Fazriyah, Supriyati, & Rahayu, 2018; Permana, Hindun, Rofi'ah, & Azizah, 2019). It also indicates how capable studentsare in comprehendingmathematical rules and principles to solve mathematical problems. Diverse and non-ordinary question designs can trigger an increase in students' creative thinking skills sincethey show how creatively each individual can be in solving math problems. The gap between schools and universities mathematics in terms of creativity requires special attention(Ayala & Manzano, 2018; Ellis & Bliuc, 2016; Hassel & Ridout, 2018; Ho & Li, 2018; K. Jones, 2020; Kahu, Nelson, & Picton, 2017; Krasilnikov & Smirnova, 2017; McCarthy, 2016; Olani, 2017; Roux, 2016; Trautwein & Bosse, 2017; van der Zanden, Denessen, Cillessen, & Meijer, 2019; van Herpen, Meeuwisse, Hofman, Severiens, & Arends, 2017; van Rooij, Jansen, & van de Grift, 2018). Discrepancies in terms of interpreting questions, using visual media, applying mathematical rules, andformulating steps to solve problems are the main focuses of this present study.

Commognitive is a combined term of commognition and cognitive(Sfard, 2007, 2008). The commognitive frameworkwhich has four components, namely, word uses, visual mediators, narratives, and routinescan be used to analyze students' works (Ioannou, 2018; Kim et al., 2019b; Lu, Zhang, & Stephens, 2019; Thoma & Nardi, 2018; Zayyadi, Nusantara, Subanji, Hidayanto, & Sulandra, 2019). In solving math problems, students often encounter cognitive conflicts and this cognitive conflict triggers a commognitive conflict(Lu et al., 2020; Nachlieli & Heyd-Metzuyanim, 2021; Pratiwi et al., 2020; Thoma & Nardi, 2017, 2018; Viirman & Nardi, 2019; ZAYYADİ, 2020; Zayyadi et al., 2019). Even though much research on commognitive conflict have been carried out, this present study is crucial since it revealed the source of the conflict. In this study, a commognitive framework was used to analyze students' works who were given math problems about dividing integers. For instance, 'What is the quotient and remainder of the division $\frac{6}{5}$?'.By applying the remainder theorem, it was found that the quotient is one and the remainder is also one. In middle school, on the topic of number division, the divisor is always less than the dividend and is never negative. Distribution problems like this are common in school mathematics.

In this study, the questions used were not commonlygiven in school mathematics, such as nonordinary, but at the university level. These non-ordinary questions are given to find out students' creativities in solving various questions and their level of understanding of the concepts given during the learning process. Theyare expected to support investigations into students' understanding of mastering the concepts. The concept of dividing numbers is that 'what is divided is equal to the divisor multiplied by the quotient plus the remainder, but the remainder must be non-negative'. These kinds of questions can trigger conflicts in their minds. The existence of confusion and doubt is an indication of cognitive conflict which triggers a commognitive conflict. Thus, indications of a commognitive conflict can be seen from the results of the analysis by using a commognitive framework. The causes of this commognitive conflict were explored by conducting interviews.

METHOD

This study aimed to find out whether or not there is a commognitive conflict that occurs instudents in solving uncommonproblems in the division of integers. The tasks given to the participants consisted of three questions;two ordinary questions and one non-ordinary question. These three questions are about the division of numbers, where there are divisions, divisors, quotients, and remainders. To get this aim, this study employed descriptive qualitative study(Cresswell, 2013). A total of 40 students of mathematics education from several universities in Indonesiawho had taken Number Theory and Basic Mathematics courses were asked to answer the questionsindividually for 30 minutes. The prospective subjects who indicated experiencing commognitive conflict were selected as the research subjects and invited toin-depth interviews. The semi-structured interviews were conducted to dig deeper into information about the causes of the cognitive conflict. The results were then analyzed by using a commognitive framework.

RESULT

Based on the results of 40 participants Mathematics education, questions number 1 and number 2 could be solved well by all participants. However, for question number 3, there was only one participant who could solve it correctly, while 39 of themhad it incorrect and thus they became the prospective subjects. The prospective subjects were those whoanswered correctlyon question number 3 and those who answered incorrectly were indicated to experience commognitive conflict. Out of the 39 potential subjects, two people whoindicated experiencing commognitive conflict in the four components were selected to have in-depth interviewsto explore the causes of commognitive conflict. The followingsare the results of the two research subjects.

Subject S1

The following are the excerpts of interviews and commognitive analysis regarding the results of S1's work related to question number 3:

Transcripts of	Commognitive Components			
interview	Word Uses	Visual Mediators	Narratives	Routines
R: How did you solve question number three? S1: I replaced -5 with its absolute number, i.e. 5 5 : 4 = 1, remains 1.	-5 digantikan dg angka absolutnya yaitu 5 5;4=1 sisa 1. Translate: -5 replaced with its absolute value 5 5:4 = 1, the remainder is 1			
	Using an absolute value from -5 to 5 should not be accepted.			
R: How did you illustrate the completion steps? S1: I used the number line to determine the quotient and remainder.		Using the number line to divide -5 by 4, goes one unit left four times and one right, as a remainder of 1 and that's incorrect.		
R: Why did you answer so?			Since -5 is negative, then	The steps that were done in

S1: At first, I was		-5/4 is also	high school
confused, but then I		negative. So,	usuallyused
answered it. The		the quotient -1	the remainder
divisor is usually		and	theorem, but
less than the		remainder is	remembered
numerator, but here		1. Even	that the
is 4 > - 5		though $-5 = -$	remainder
R: Are you sure		1(4) + (-1) and	must be non-
about that?		not 1.	negative, the
S1: At first, I was		There was a	solution step
confusedand		difference	wasincorrect.
doubtfulbut yes, I		with previous	
answered that way.		knowledge,	
R: What was your		namely the	
reason?		divisorwas	
S1: By using the		always	
remainder		smaller than	
theorem, which is		the numerator	
the numerator		and the	
equals to the		remainder is	
quotient times the		non negative.	
divisor plus the		That's	
remainder.		incorrect.	
R: Does it mean			
thatthe remainder is			
1?			
S1: Yes, but I am			
not sure about that.			
R: Why?			
S1: Because the			
divisor is greater			
than the numerator.			
The remainder is 1,			
because the			
remainder cannot			
be negative even			
though with the			
remainder theorem,			
-5 = -1(4) + (-1) and			
not 1			

S1 replaced -5 with its absolute value, namely 5. There was a difference in meaning between the number -5 and 5. It can be said that there was a commognitive conflict in the words use component. Illustratedby a number line to solve the problem, there was an error when divided -5 by 4, whichtaking four steps to the left each one unit, which are depicted by arrows pointing to the left and one step to the right by one unit, illustrated by arrow to the right. It showed that there was a commognitive conflict in the visual component of the mediator since the direction of the arrow should remain to the left. S1 considered that the remainder of the division to be 1 (one) because the direction was to the right. S1 believed that the remainder of the division must be non-negative. But -5 was negative, so -5/4 was also negative. Thus, the quotient was -1 and the remainder was 1. Even though -5 = -1(4) + (-1) and not 1. S1 experienced confusions and doubts because of the difference

in the results of rechecking. There was a difference with previous knowledge that the numerator and the divisor should be positive and the divisor is always smaller than the numerator, although the remainder is non-negative. The steps taken by S1 to solve the questions were also incorrect, and the clarification through interviews showed that the narratives and routines components contained of commognitive conflict.

Subject S2

The followings are excerpts of interviews and cognitive analysis regarding the results of S2's work related to question number 3.

Transcripts of	Commognitive Components			
interview	Word Uses	Visual Mediators	Narratives	Routines
R: How did you solve the question number three? S2: I wrote -5 : 4 = -1 remain -1	-5 = -1(4) + -1 Hasil bagi <u>-1</u> sisa -1 Translate: -5 = -1(4) + -1 The quotient is -1 and the remainder is -1 The quotient was -1 and the remainder was -1, it			
R: How did you illustrate your steps? S2: I illustrated -5 as 5 groups that -1 for each, then divided by 4, and related to 4 other groups and remained one group. So, the quotient was -1 and the remainder was -1, one group of -1.	was incorrect.	-5 4 4 Drew 5 groups that each represent -1, then paired them with 4 other groups. There was one group that didnot have a partner. So, the quotient was -1 and the remainder was -1, and that was incorrect.		
R: What was your reason for answering that? S2: To be honest I was confused, but, Ijust answered it. Usually, the divisor and the numeratorwerepositive numbers and the divisor is always usually smaller than the numerator but not in this case. R: Are you sure about your answer? S2: Not sure, I was so			There is a difference in the concept of dividing integers with previous knowledge, namely the numerator and the divisor are positive numbers and the	There were steps that are usually done in school andthat is incorrect.

confused and doubtful.		remainder	
Butyes, I just		of the	
answered. It like that.		division is	
By using the remainder		not	
theorem, that the		negative.	
numerator equals to the		The divisor	
quotient times the		whose value	
divisor plus the		is greater	
remainder.		than the	
R: Does it mean that		divisor is	
the quotient was -1 and		also	
the remainder was -1?		different	
S2: Yes, but I still have		from the	
a doubt on my answer.		previous	
Because the divisor		knowledge.	
and the numerator are			
usually positive and in			
this question the			
divisor is greater than			
the numerator. And			
also, the remainder is			
negative.			
R: Are you familiar			
with the concept of			
division of integers?			
S2: I remember it like			
that which was			
explained by my			
previous lecturer. In			
the past, when I was at			
high school, my			
teacher's explanation			
was the same.			

S2 divided -5 by 4 stated that the quotient was -1 and the remainder was -1. This remainder wasincorrectsincethe remainder must be non-negative. S2 illustratedthe division f -5 by 4, created 5 groups that each of them represents -1. Each group was paired with four other groups and remained one group. Therefore, S2 stated that the quotient was -1 and the remainder was -1. There was a commognitive conflict in the visual mediators that the remaining one group represents -1. S2 started to feel doubtful and confused sincethe remainderwas negative. Usually, the divisor and the numerator are positive numbers and the divisor is usually smaller than the numerator but not in this case. This raised a conflict in S2's mind, indicated a commognitive conflict in the previous theorythat 'what is numerator and divisor were positive number, and numerator is usually greater than the divisor'. Hence, when the divisor is greater than numerator, it emergeddoubts and confusion in S2. There was a difference in the concept of division integers with previous knowledge that has been obtained from school of mathematics and previous lecturers so that triggered a commognitive conflict.

DISCUSSION

The gap between school and university mathematics can lead to commognitive conflict in terms of the four commognitive components, namely word uses, visual mediators, narratives, and routines(Kim et al., 2019a; Pratiwi et al., 2020; Thoma & Nardi, 2018; Zayyadi et al., 2019). It means that the students' creativities in

practicing various questions and critical thinking skills are demanded at a higher level at the university level. Critical thinking is required to be mastered by students so that they are able to understand and analyze questions properly(Chikiwa & Schäfer, 2018; Firdaus et al., 2015). It can be said that lack of critical thinking skills leads to errors in understanding the questions given. Besides, it can raise controversy in solving non-ordinaryproblems(Maharani & Subanji, 2018; Mustafa, Nusantara, Subanji, & Irawati, 2016; Syamsuri, Purwanto, Subanji, & Irawati, 2017). Errors in interpreting words can trigger commognitive conflicts in the word uses componentin line with the opinions in the previous studies (Kahu et al., 2017; López, Cerveró, Rodríguez, Félix, & Esteban, 2013). According to Hassel & Ridout(2018), Kahu et al.(2017), and López et al.(2013), this condition can affect student learning achievement, especially in the first year of lectures. The research subject interpreted the division as occurring in two non-negative numbers which triggers this conflict. The variety of questions received in secondary schools still refers to examples of questions given by the teacher and is very weak in developing the concept.

The problem-solvingsteps used by the subjects reflect that their creativities in exploring their knowledge is still low as cited in Gunnell, Mosewich, McEwen, Eklund, & Crocker(2017), Hassel & Ridout(2018), Kahu et al.(2017), and van Herpen et al.(2017). This can be seen when they encounter nonordinaryquestions and experience cognitive conflict which triggers commognitive conflict which was agreed byKim et al.(2019a), Pratiwi et al.(2020), Thoma & Nardi(2017, 2018). The non-ordinary questions regarding the division of integers also use visual media for representing numbers (visual mediators), including number lines or other media. Subject used the number line but ignored the rule that the remainder must be non-negative. In ordinary questions, the number line media is very good to apply, butit will be difficult to apply this methodin non-ordinaryquestions. This can be seen in the two subjects who used visual media in solving the non-ordinary questions given but they ignored the rule that the remainder of the division must be non-negative. The subject's knowledge of the remainder theorem, if they understand, can be used to answer non-ordinary questions well. However, the knowledge they received in high school and held so firmly that the divisor is always less than the divide, and the two are never negative. Both subjects stated that the results of the division and the students obtained were correct. But they did not realize that the remainder of the division must be negative. This contradicts their previous knowledge and thus triggering a cognitive conflict in the narrative component, in line with Thoma & Nardi(2017), Kim et al.(2019a), Pratiwi et al.(2020), and Nachlieli & Heyd-Metzuyanim(2021).

After getting complete information about the rules for dividing integers, the subjects finally accepted this new theoryeven though they were still confused and doubtful about the rules before. The concept that remainder is always non-negative has never been accepted by them before. This went against the knowledge inherent in their mind, thus the unusual problem was able to reveal this. At first, they were confused and doubtful, which is a sign of conflict, but in the end they are able to accept the new concept. Commognitive conflict occurs in the narratives component, where there is conflict between the previous and the new knowledge they received and this can be explored when conducting interviews, in the line with Kim et al.(2019a), Pratiwi et al.(2020), Nachlieli & Heyd-Metzuyanim(2021). In the routines component, the steps for solving questions based on their knowledge in high school are different from those in university. This is in the line with Zayyadi et al.(2019), Pratiwi et al.(2020). Commognitive conflicts in the routines component can be revealed during interviews when the subject explained what steps and rules they used to solve non-ordinary questions. The steps they took to solve ordinary questions and their lack of understanding of the rules used caused commognitive conflicts in the routines components. In this study, non-ordinary questions were able to reveal the source of the commognitive conflict in the division of integers based on the four commognitive components.

CONCLUSION

Ordinary questions encountered in high school can be developed into non-ordinary questions and obtained at the university. Students' creativities to develop the questions they have obtained and a strong understanding of the concepts previously obtained support this study. On the other hand, a weak understanding of the concept of sharing that they know causes confusion and hesitation when encountering non-ordinary

questions, which causes cognitive conflict and thentriggers commognitive conflict. Commognitive conflict in the words use component occurs because of the difference between their previous knowledge and the new rules they received. The remainder of the division that is negative triggers errors and there is a cognitive conflict that emerges the commognitive conflict. This encourages errors in the problem-solving steps so that the routines component also experiences conflicts. Weak understanding of previous knowledge in secondary schools, that are different from universities, triggers commognitive conflicts in the narratives component. As a result, increasing students' understanding and creativitymustbe done to reduce the occurrence of cognitive conflicts in other materials as well.

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